

# Synthesis of Embedded Control Software

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Caltech, Control and Dynamical Systems

Papers, slides, notes, software tools at  
[www.cds.caltech.edu/~UTopcu](http://www.cds.caltech.edu/~UTopcu)

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# Synthesis of Embedded Control Software

Joint work with  
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(MIT, Singapore) (Caltech) (Caltech)

## Outline

- Setup
- Receding horizon temporal logic synthesis
- Vehicle management systems
- Distributed synthesis

# How to automatically design control protocols, that...

Handle mixture of discrete and continuous decision-making

Account for both high-level specs and low-level dynamics

Ensure proper response to external events in real-time,

... with "correctness certificates"?

# How to "automatically" design control protocols that...

- Handle mixture of discrete and continuous decision-making
- Account for both high-level specs and low-level dynamics
- Ensure proper response to external events in real-time

## Autonomous driving



## Vehicle management

federated → IMA

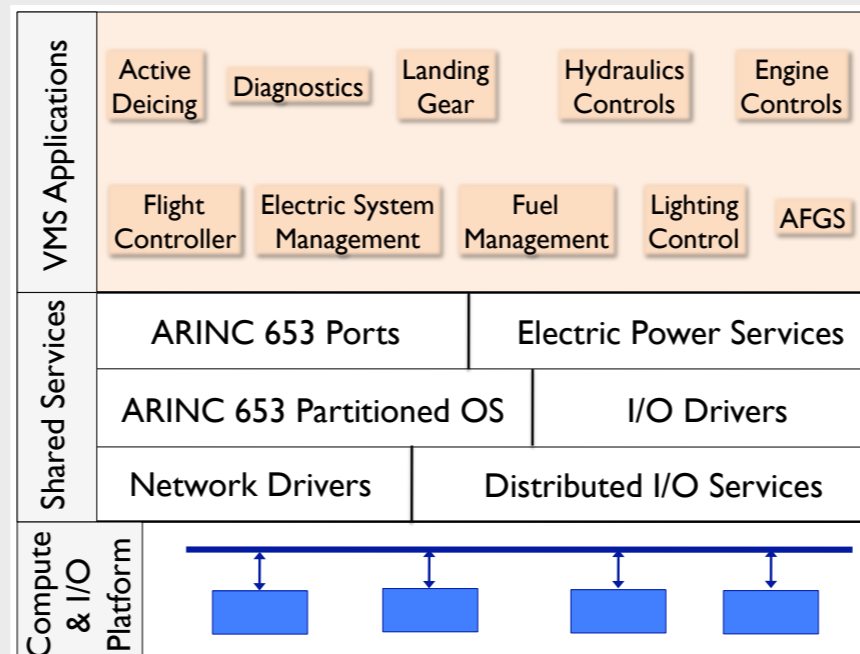
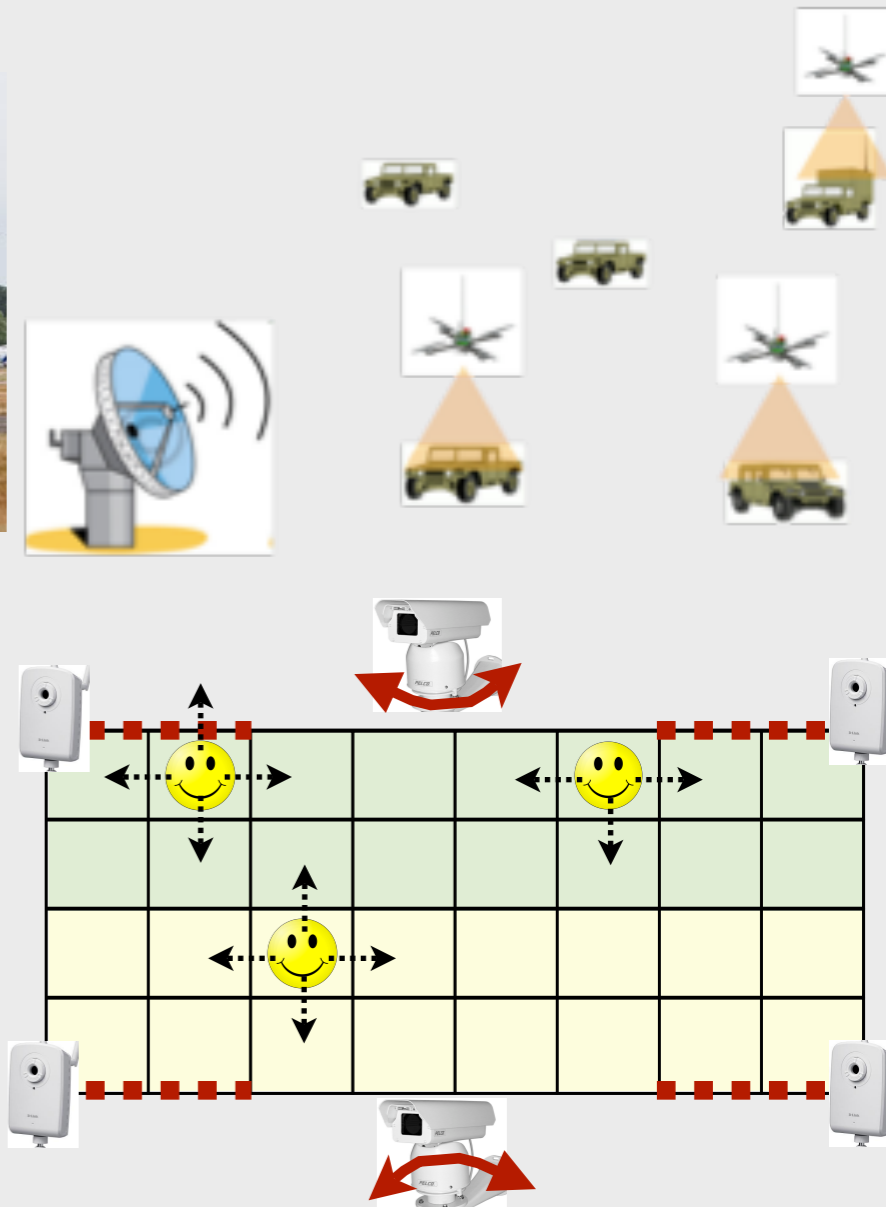


Figure – regenerated from a similar figure by W.P.Kinahan, Sikorsky Aircraft

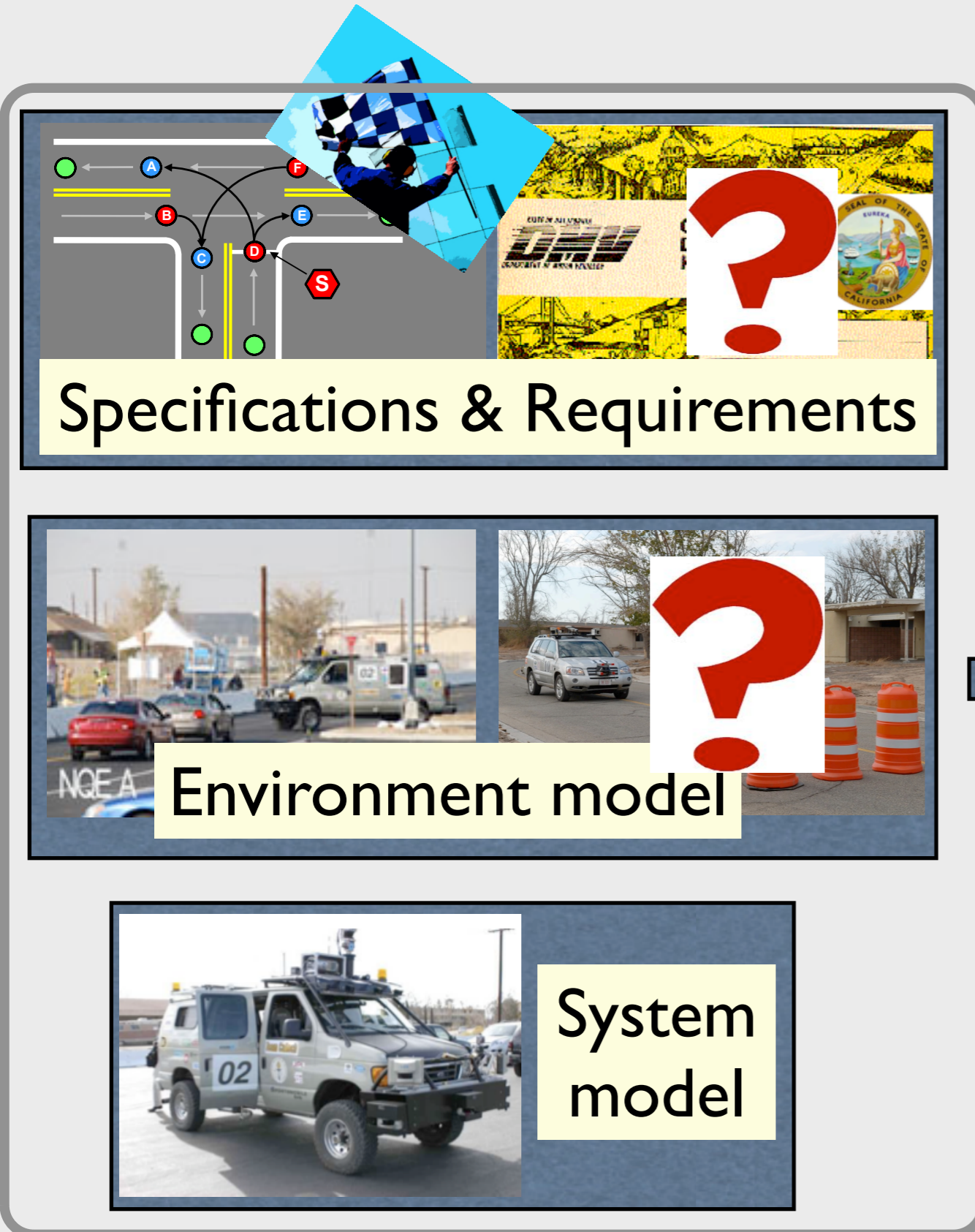
## Active surveillance



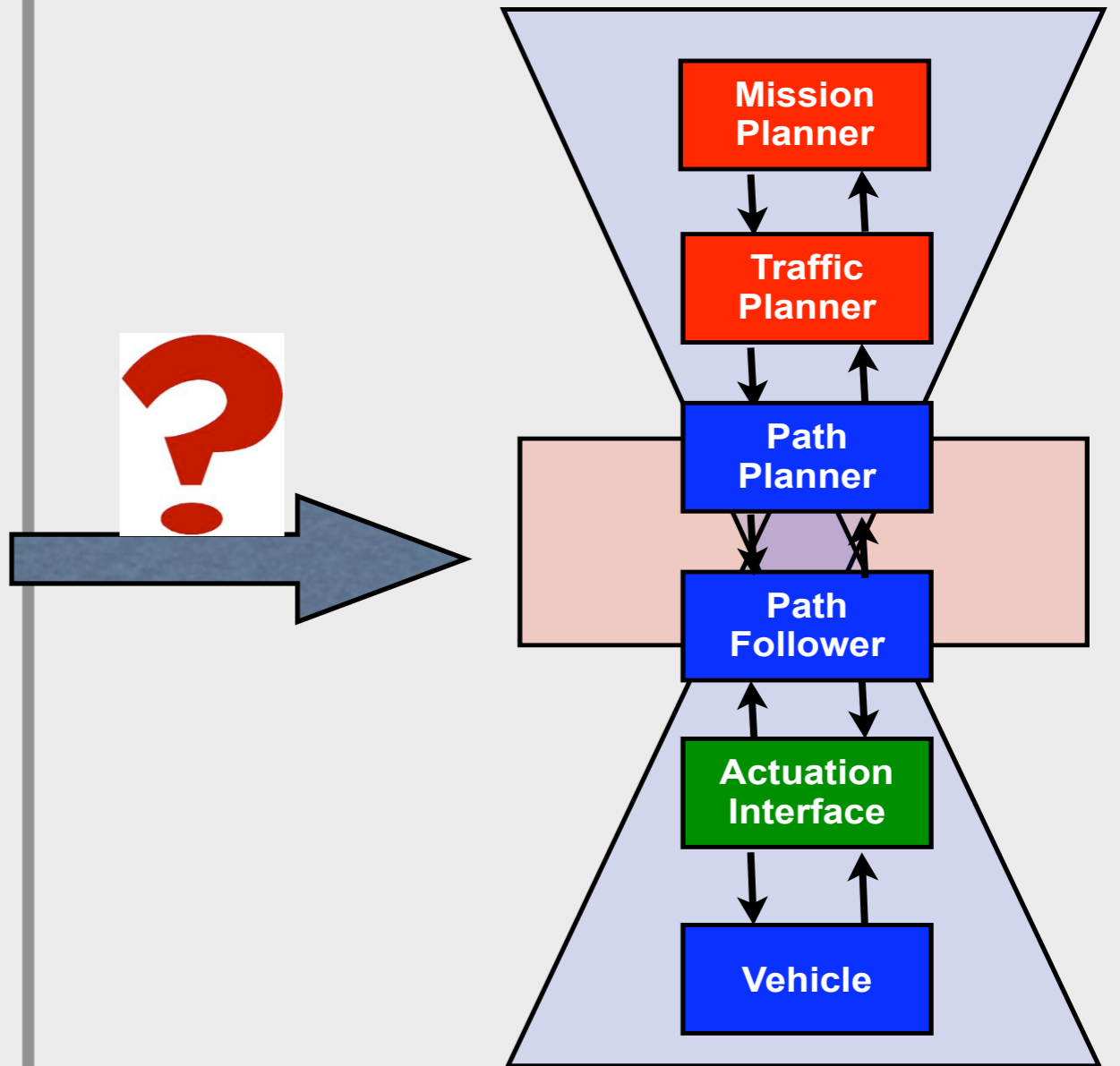
Synthesis of Embedded Control Software



# Inputs & Outputs



## Alice's planning stack



# Specifying behavior with linear temporal logic (LTL)

Extends **propositional logic** with **temporal operators**

$$\left( \begin{array}{l} \wedge \text{ (and), } \vee \text{ (or),} \\ \rightarrow \text{ (implies), } ! \text{ (not),} \end{array} \right) + \left( \begin{array}{l} \diamond \text{ (eventually), } \square \text{ (always),} \\ \mathcal{U} \text{ (until).} \end{array} \right)$$

- Allows to reason about infinite sequences of states
  - state: snapshot of values of all variables (environment+system)
- Specifications (formulas) describe sets of allowable behavior
  - safety specs: what actions are allowed
  - fairness: when an action can be taken (e.g., infinitely often)
- No strict notion of time. Just ordering of events.

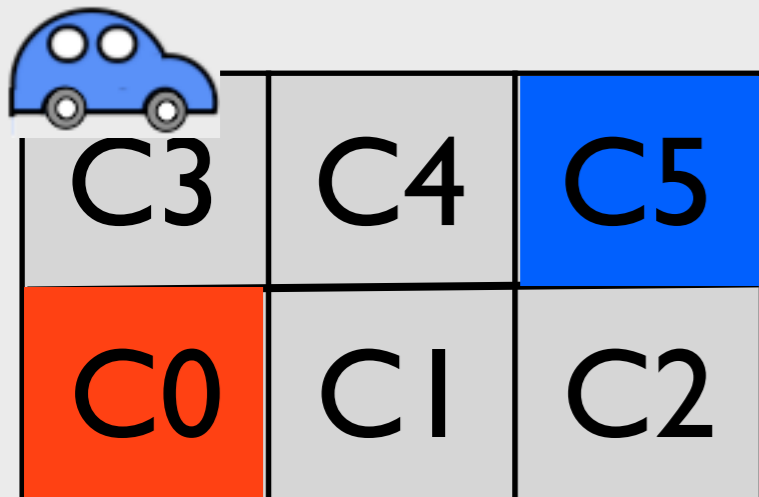
# Compose to specify interesting behavior

$p \rightarrow \diamond q \equiv p$  implies eventually  $q$  ( $\sim$  response)

$\square \diamond p \equiv$  always eventually  $p$  ( $\sim$  progress)

$\diamond \square p \equiv$  eventually always  $p$  ( $\sim$  stability)

$p \rightarrow q \mathcal{U} r \equiv p$  implies  $q$  until  $r$  ( $\sim$  precedence)



## Desired properties:

- Visit C5 infinitely often.
- Whenever a park signal is received go to C0.

## Environment assumption:

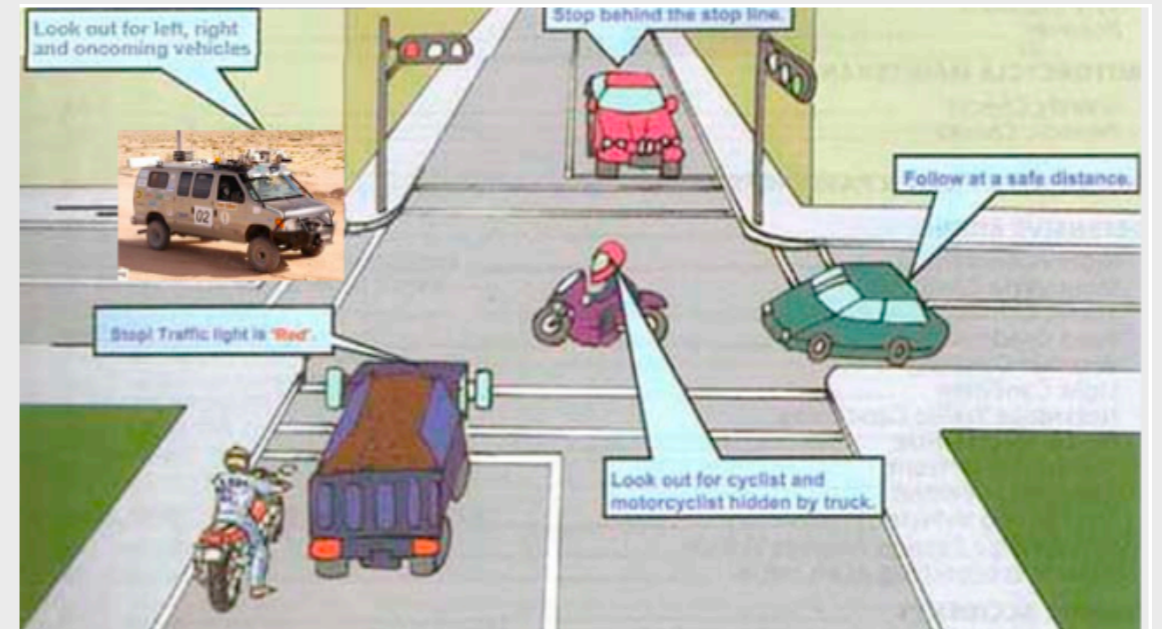
- Park signal is not received infinitely often.

$$\square \diamond (!\text{park}) \rightarrow \{ \square \diamond (s \in C5) \wedge \square (\text{park} \rightarrow \diamond (s \in C0)) \}$$

# Sample Specifications

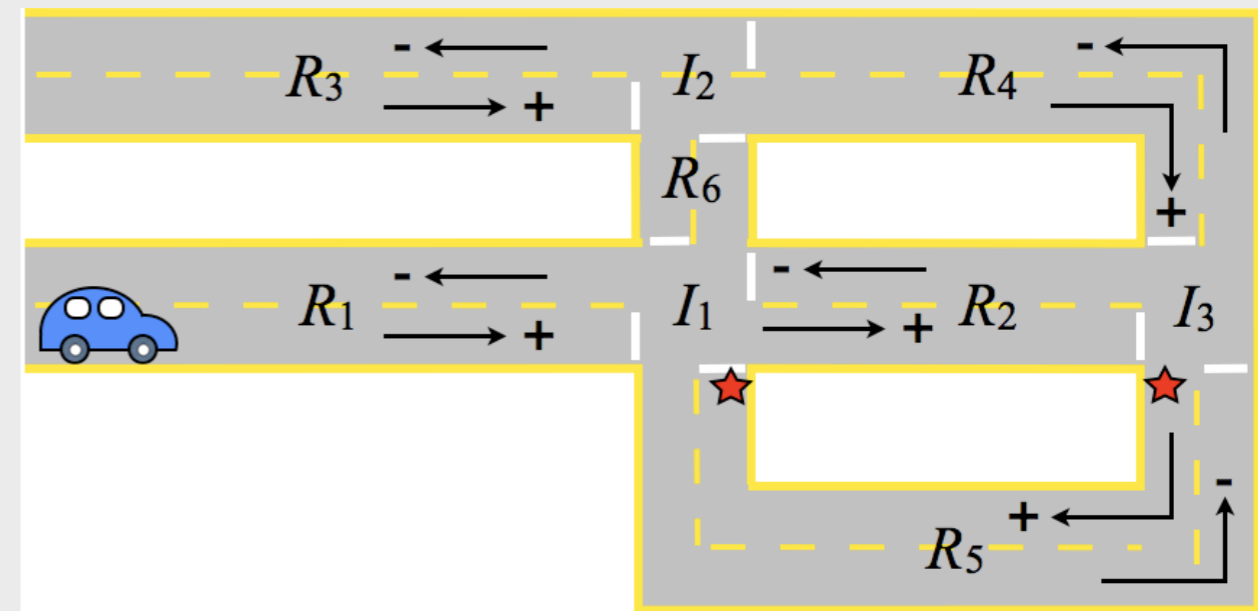
## Traffic rules:

- No collision
- Stay in travel lane unless blocked
- Go through an intersection only when it is clear



## Environment Assumptions:

- No road blockage
- Limited sensing range
- Detect obstacles before too late
- Obstacles close to the car do not disappear
- Each intersection is clear infinitely often
- Vicinity of ★'s is obstacle-free infinitely often

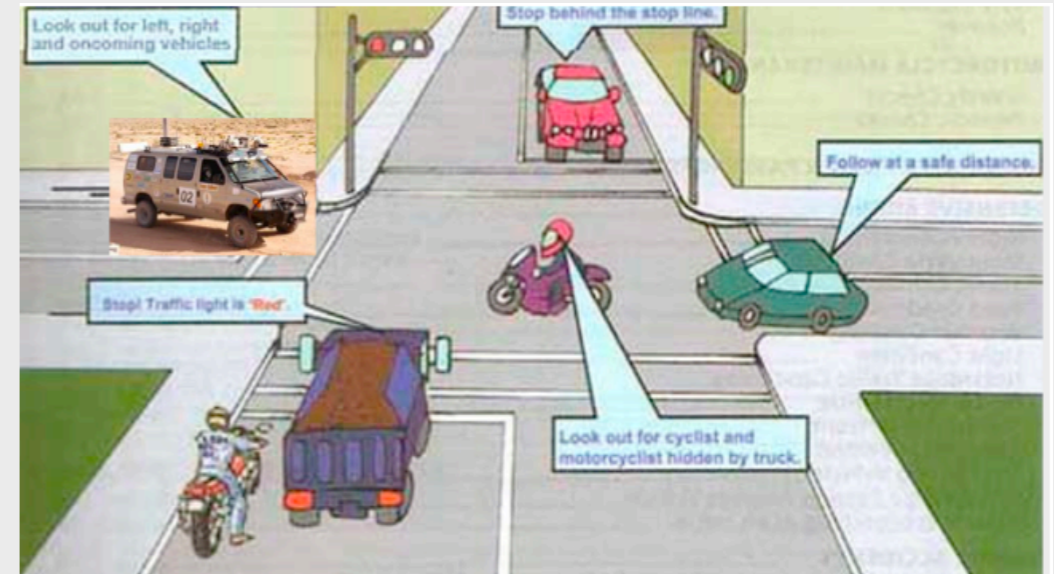


Goals: Go through ★'s infinitely often



# Temporal Logic Planning

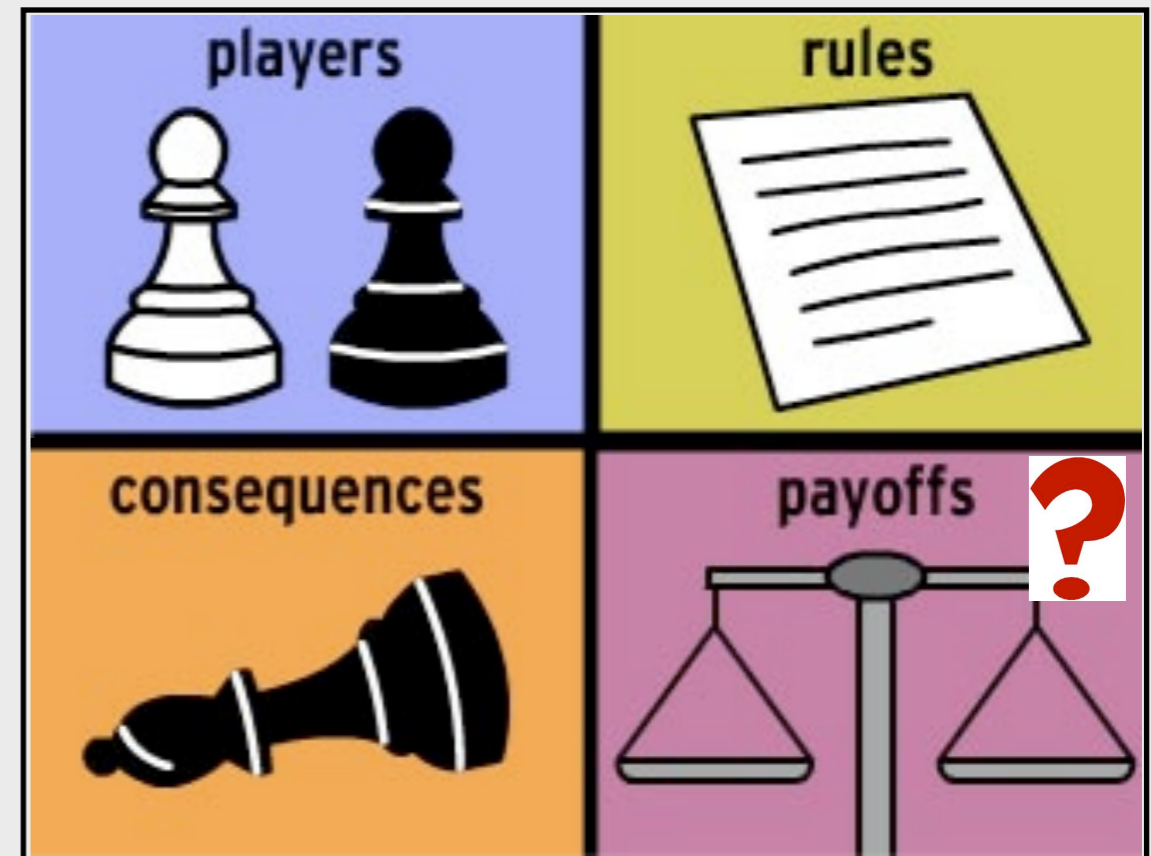
Construct a control protocol such that the system satisfies



$$\varphi_{init} \wedge \varphi_{env} \rightarrow \varphi_{safety} \wedge \varphi_{goal}$$

## Game interpretation:

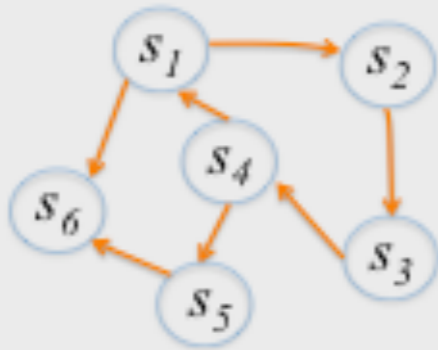
A game between system & environment



# Discrete Synthesis

- states = (system, environment)
- all executions satisfy the spec's

Finite  
Transition  
System



Specifications

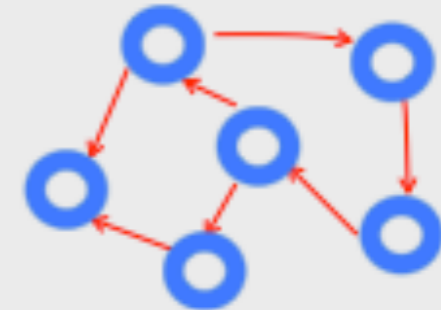
$$\varphi_{init} \wedge \varphi_{env}$$

$$\rightarrow \varphi_{safety} \wedge \varphi_{goal}$$

Discrete  
Synthesis  
Tool

Piterman, Pnueli, Sa'ar

Discrete  
Planner

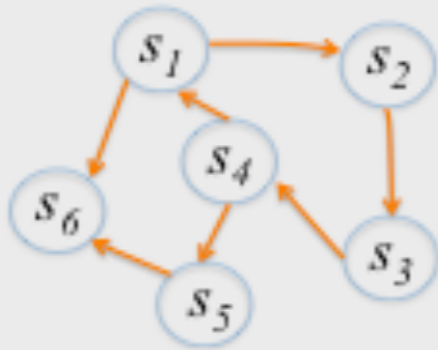




# Discrete Synthesis

- states = (system, environment)
- all executions satisfy the spec's

Finite Transition System



Specifications

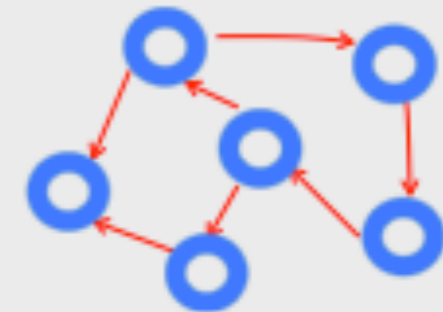
$$\varphi_{init} \wedge \varphi_{env}$$

$$\rightarrow \varphi_{safety} \wedge \varphi_{goal}$$

Discrete Synthesis Tool

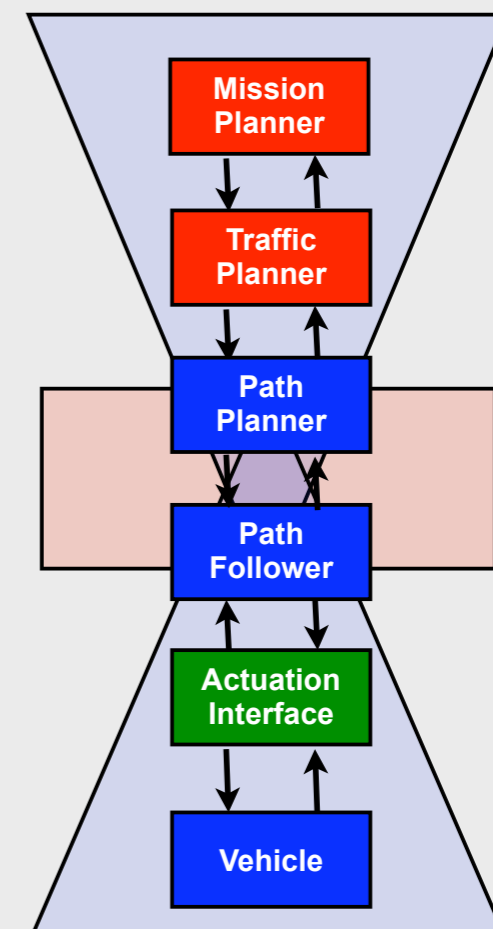
Piterman, Pnueli, Sa'ar

Discrete Planner



Most systems of interest feature interaction between

- physical components
- computing, communication,...

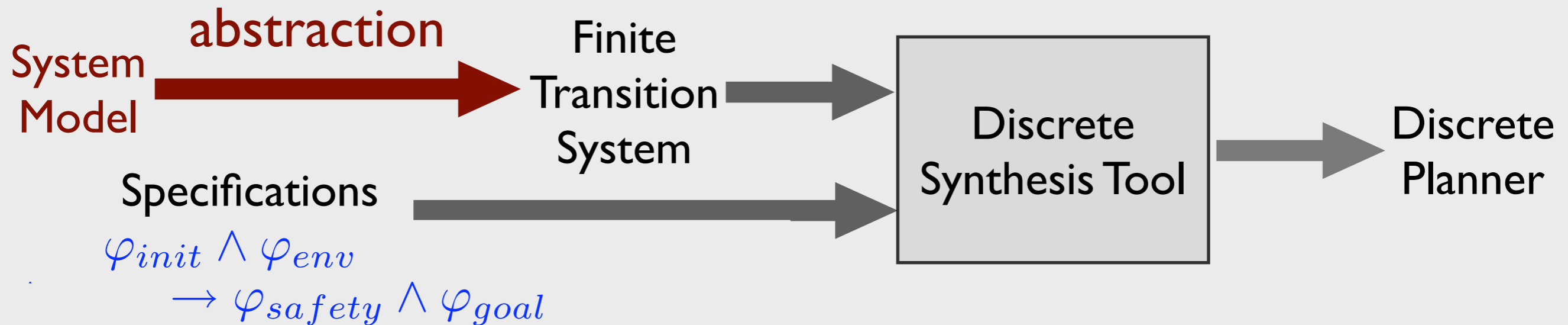


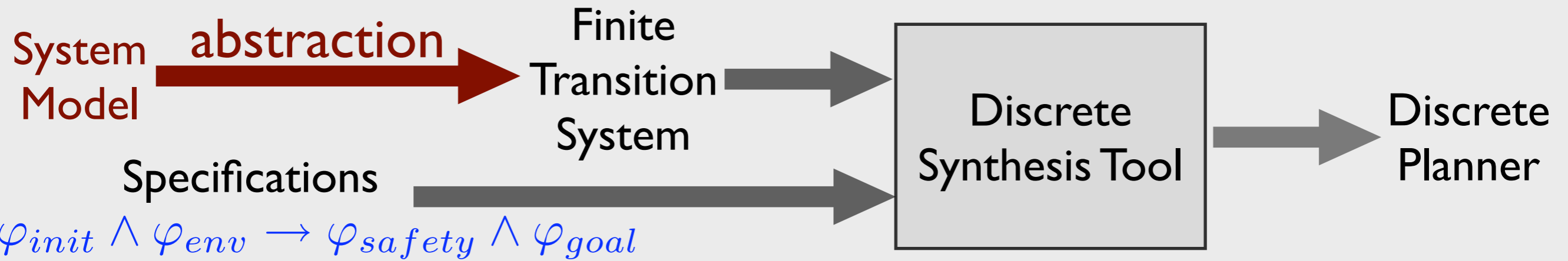
# Incorporating Continuous Dynamics

## System model:

$$\xi(t + 1) = f(\xi(t), w(t), u(t))$$

- bounded control authority  $u \in \mathcal{U}$
  - external disturbances  $w \in \mathcal{W}$
- + modeling uncertainties

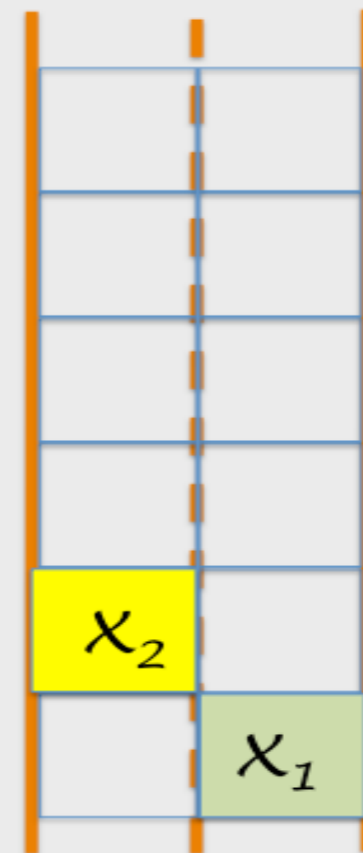


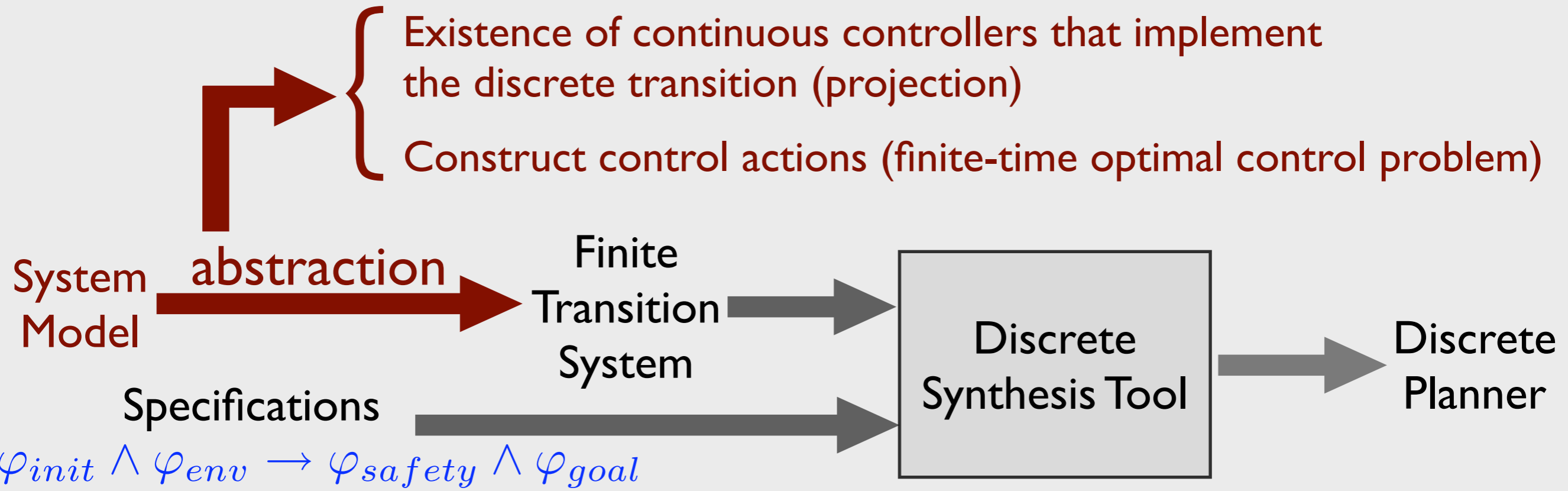


Starting with a proposition preserving partition:

Control-oriented tools to account for ...

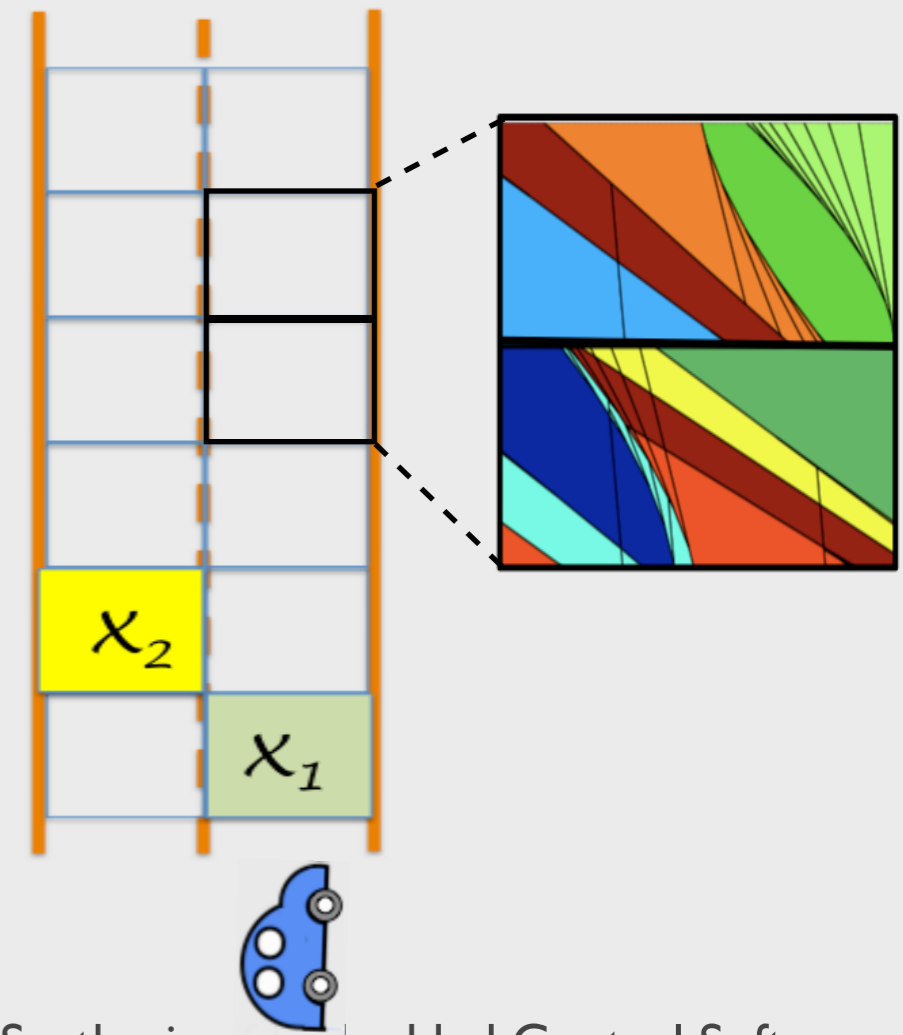
- Finite-time reachability to determine discrete transitions





Starting with a proposition preserving partition:

- Control-oriented tools to account for ...
- Finite-time reachability to determine discrete transitions
  - Refine the partition to increase the number of valid discrete transitions



# Hierarchical Control Architecture

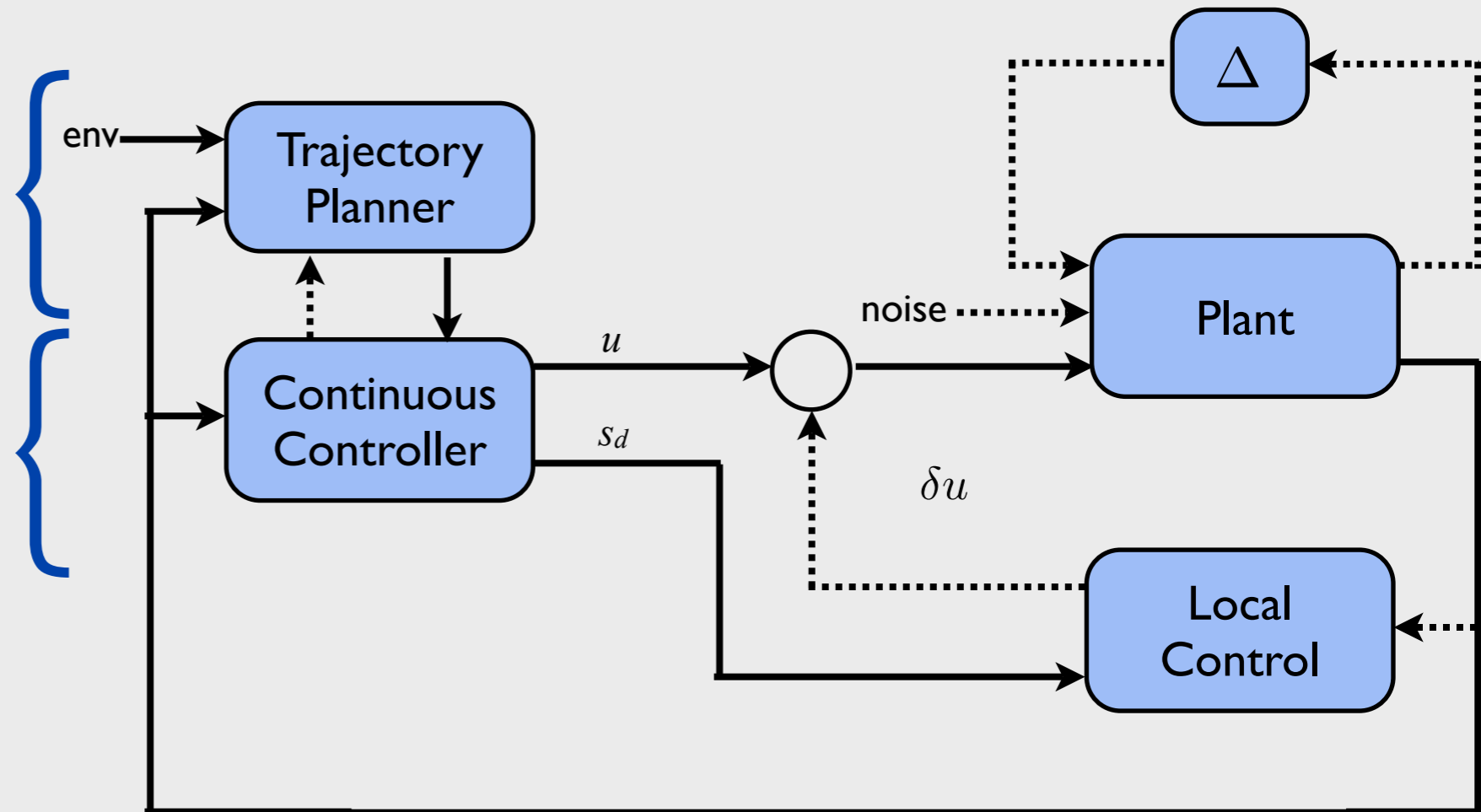
Discrete planner ensures that the spec is satisfied

+

Continuous controller implements

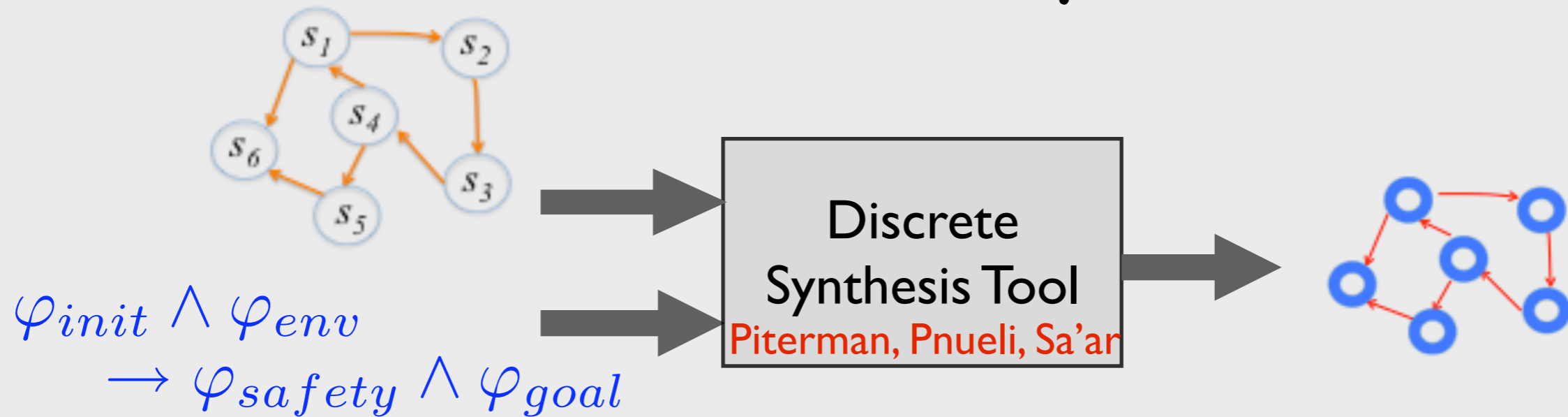
the discrete plan (handles low-level

dynamics & constraints)



When put together,  
guaranteed to work "correctly."

# More on the Discrete Synthesis Tool...



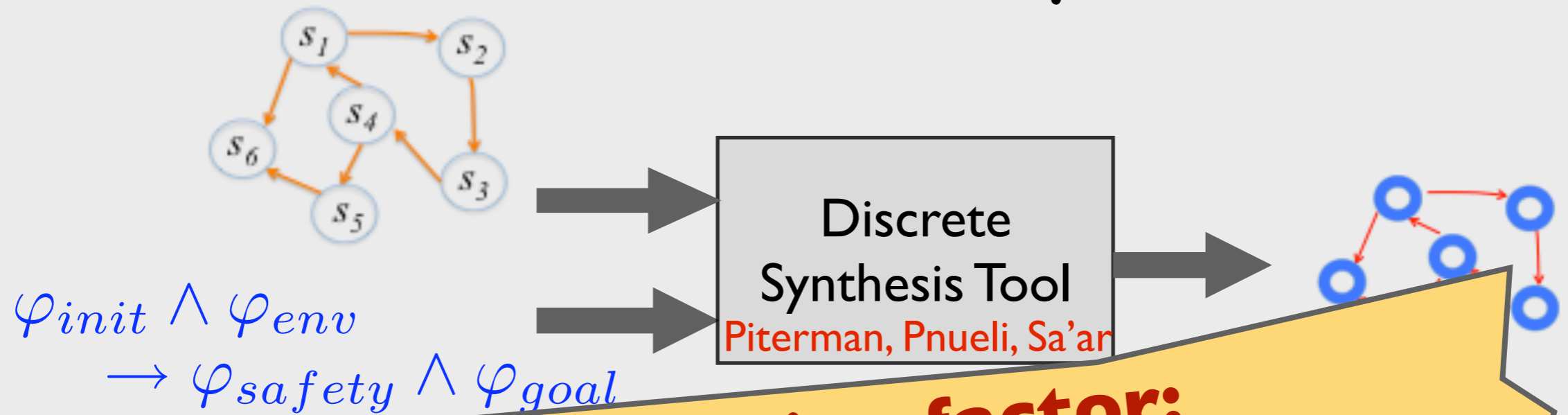
- General LTL synthesis is hard
- An expressive subclass (GR(1) games) takes “polynomial” effort

$$\bigwedge_{i=1}^m \square \diamond p_i^e \rightarrow \bigwedge_{j=1}^n \square \diamond q_j^s$$

- Based on fixpoint computations & BDDs
- Implemented in JTLV



# More on the Discrete Synthesis Tool...



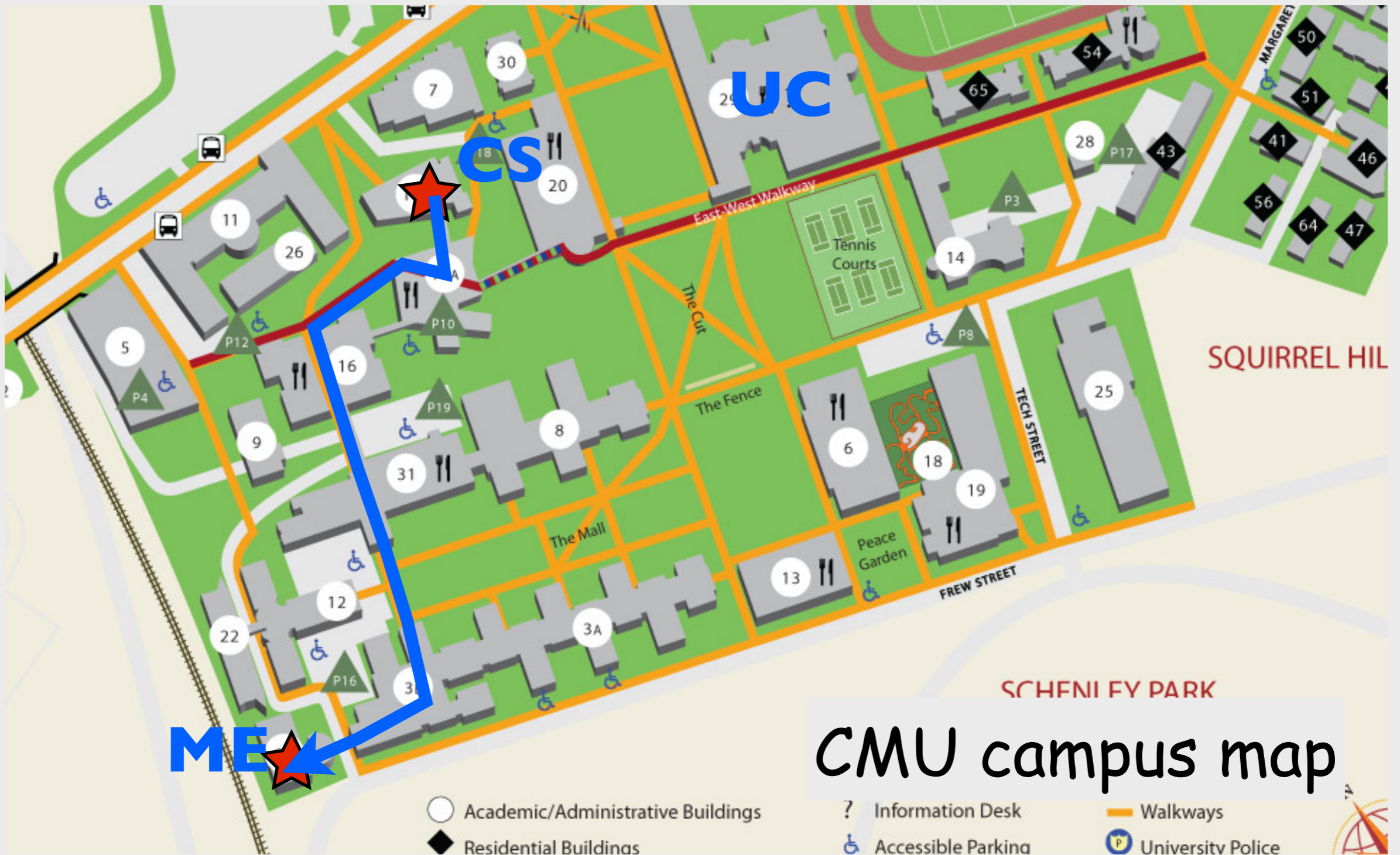
## A limiting factor:

synthesis procedure considers all possible environment behaviors

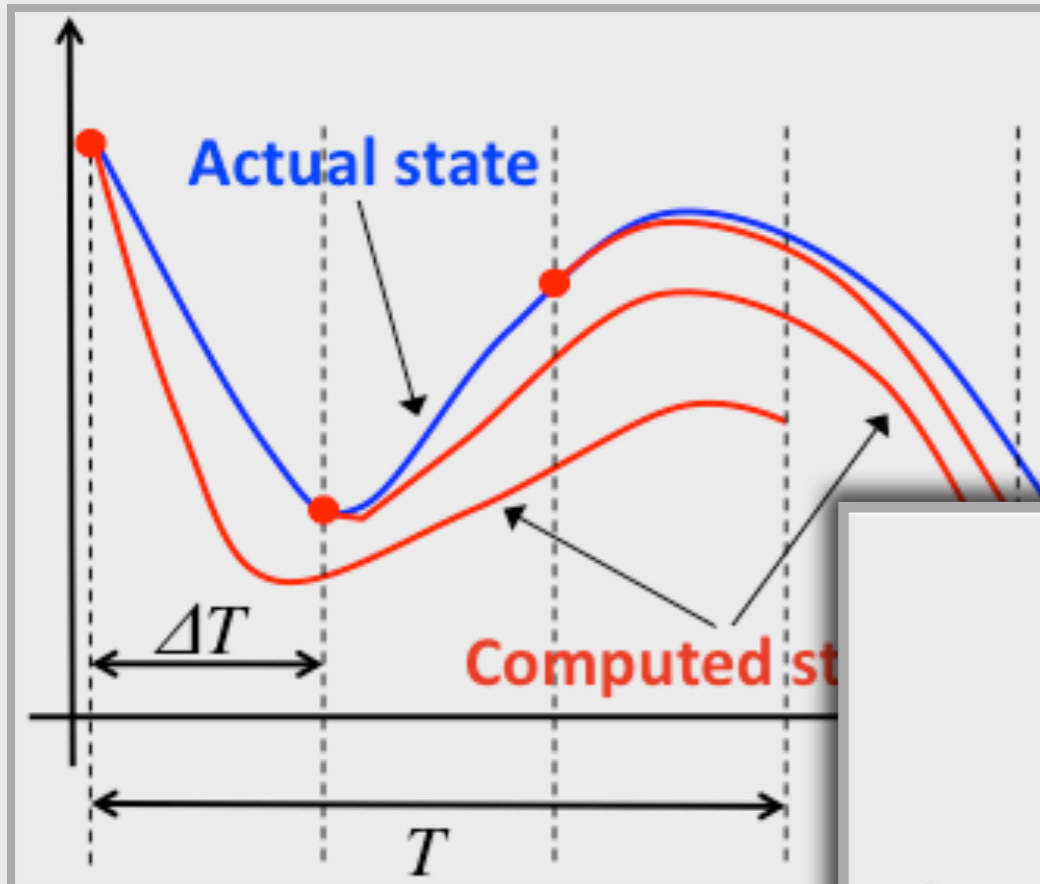
class (GR(1) games) takes "polynomial" effort

$$\bigwedge_{i=1}^m \square \diamond p_i^e \rightarrow \bigwedge_{j=1}^n \square \diamond q_j^s$$

- Based on fixpoint computations & BDDs
- Implemented in JTLV

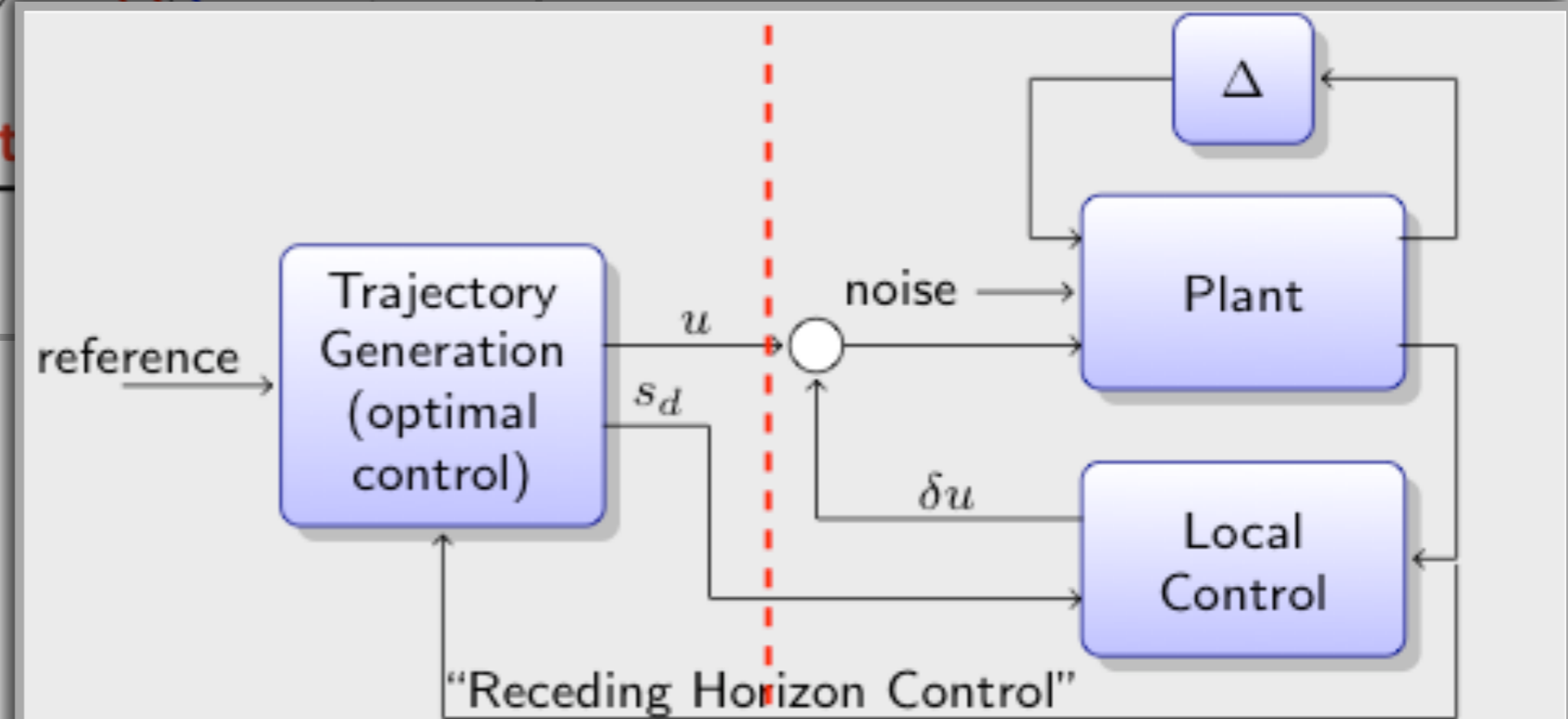


# Recall: Receding Horizon Control (RHC)



RHC can destabilize if not done properly!

Receding horizon,  
temporal logic  
planning?



**Account for:**

- global nonlinearities
- constraints
- high level objectives

**Handle:**

- uncertainties
- small scale (fast) disturbances



# Receding Horizon for LTL Synthesis

$$(\varphi_{init} \wedge \varphi_{env}) \rightarrow (\varphi_{safety} \wedge \varphi_{goal})$$

$(\{\mathcal{W}_j\}, \preceq_{\varphi_g})$  partial order covering system states

A mapping  $\mathcal{F}$  such that

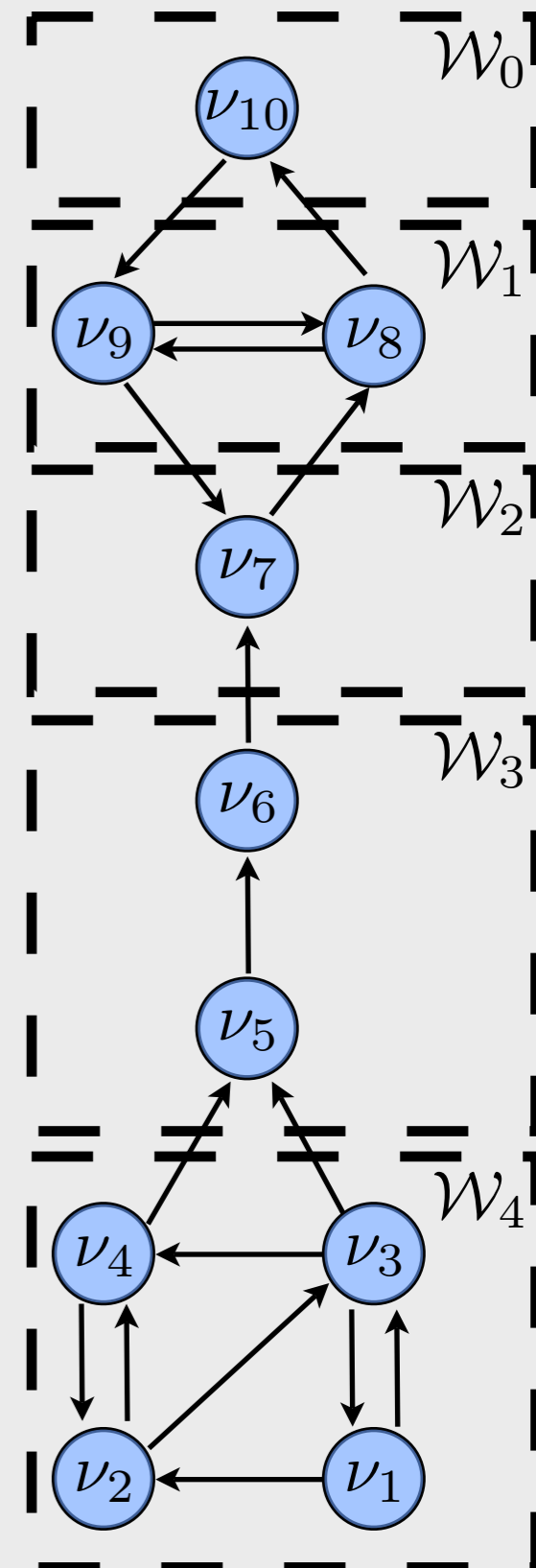
$$\mathcal{F}(\mathcal{W}_j) \prec_{\varphi_g} \mathcal{W}_j \text{ for } j \neq 0 \ \& \ \mathcal{F}(\mathcal{W}_0) = \mathcal{W}_0$$

$\Phi$ , a propositional formula such that

For each  $j$ , there exists a short-horizon controller that realizes

$$\left( (\xi \in \mathcal{W}_j) \wedge \Phi \wedge \varphi_{env}^j \right) \rightarrow \left( \varphi_{safety}^j \wedge \square \diamond (\xi \in \mathcal{F}(\mathcal{W}_j)) \wedge \square \Phi \right)$$

**Theorem:** When the system state is in  $\mathcal{W}_j$ , implement the corresponding short-horizon controller. Then, the “global” spec’s hold.

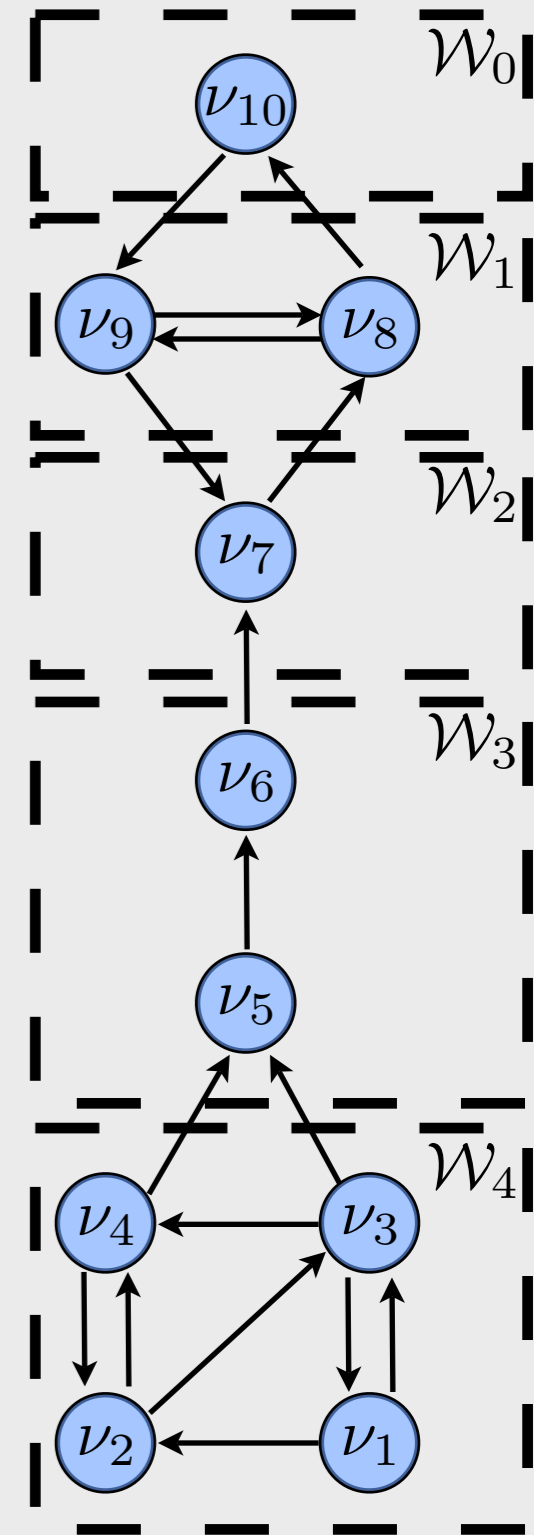


WTM@HSCCI0  
WTM@ITAC(s)

# What is $\Phi$ ?

- Receding horizon invariant, a propositional formula
- Used to exclude the initial states that render synthesis infeasible, e.g.,
  - States from which a collision is unavoidable
- Given partial order and  $\mathcal{F}$ , computation of the invariant can be automated.

- Check realizability
- If realizable, done.
- If not,
  - collect violating initiation conditions
  - negate and put in  $\Phi$
- Repeat.



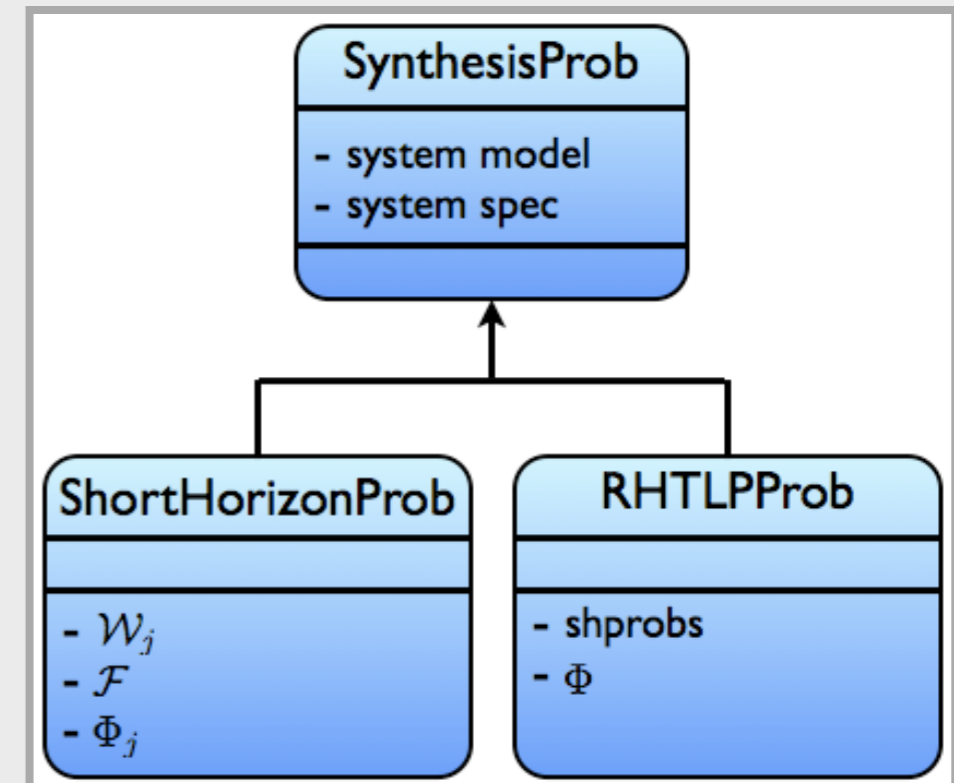
# TuLiP automates...

- Proposition preserving partitioning
- Abstraction
- Given partial order, compute an invariant (if exists)
- Verify that all conditions for applying the receding horizon strategy are satisfied
- Create short-horizon problems and implement the receding horizon strategy
- Interface to the synthesis tool
- Compute counter-examples
- Simulate the resulting strategy



A software toolbox for  
receding horizon  
Temporal Logic Planning

[www.cds.caltech.edu/tulip](http://www.cds.caltech.edu/tulip)



WTOXM@HSCC11(s)

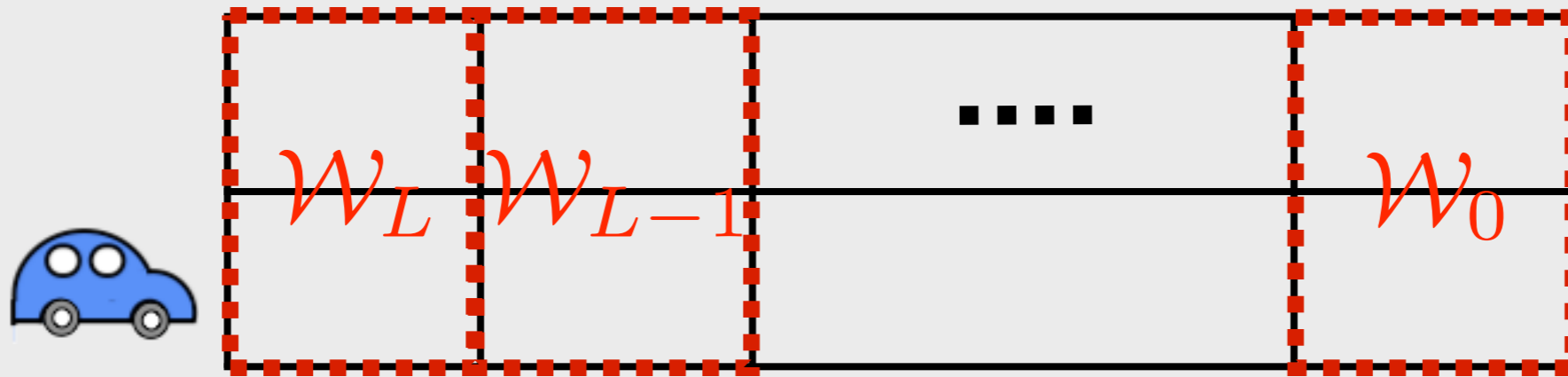
Synthesis of Embedded Control Software



# How to come up with partial order?

- Problem-dependent
- Currently requires user guidance

## Simple example



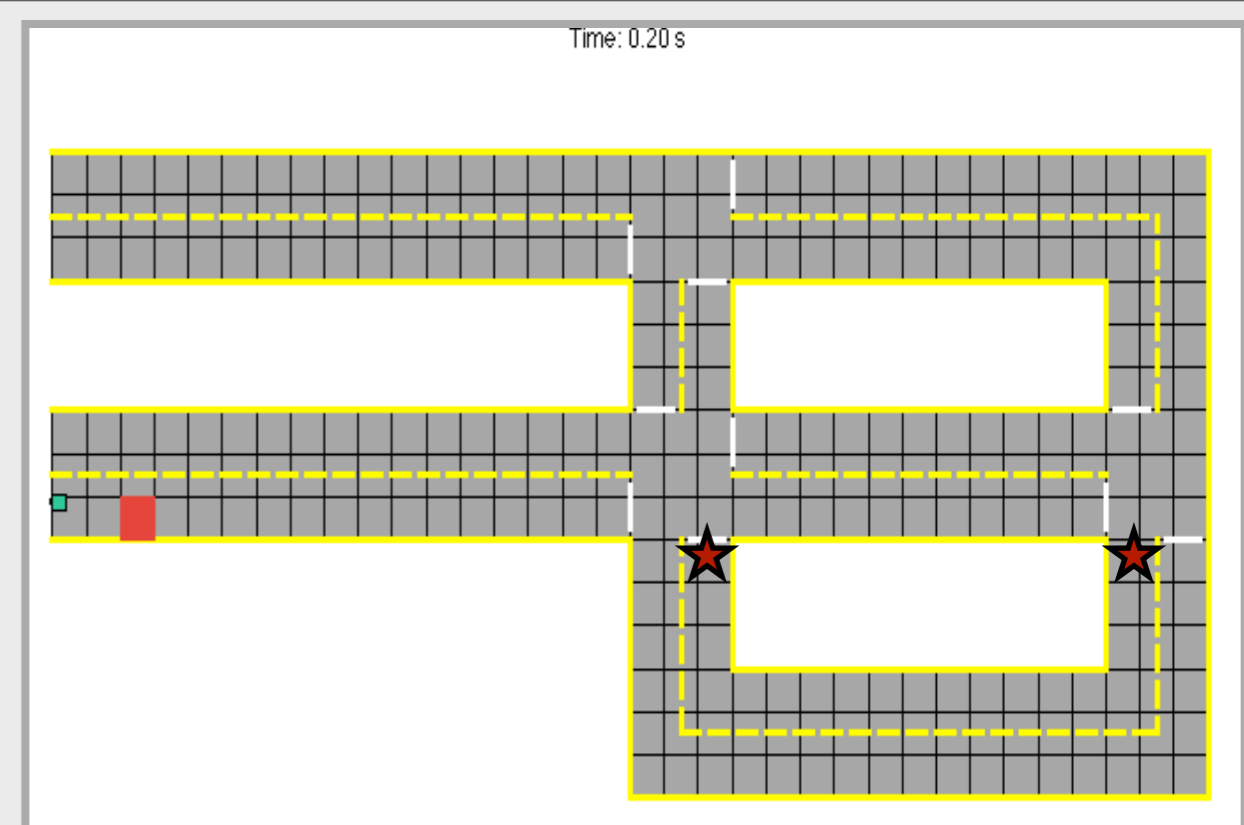
$$\mathcal{W}_0 \prec \dots \prec \mathcal{W}_{L-1} \prec \mathcal{W}_L$$

$$\mathcal{F}(\mathcal{W}_j) = \mathcal{W}_{j-2}, \quad j \geq 2$$

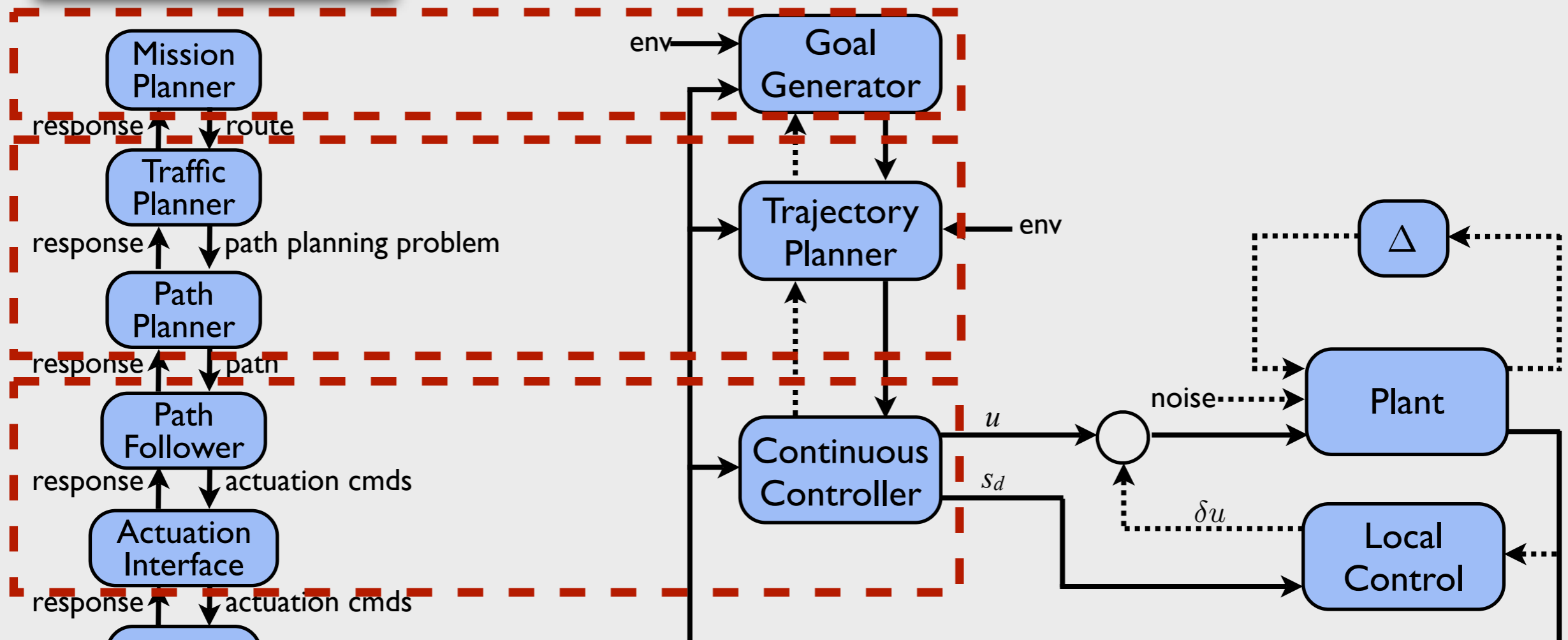
$$\mathcal{F}(\mathcal{W}_j) = \mathcal{W}_0, \quad j < 2$$

# How to come up with partial order?

In some problems, it naturally pops up.

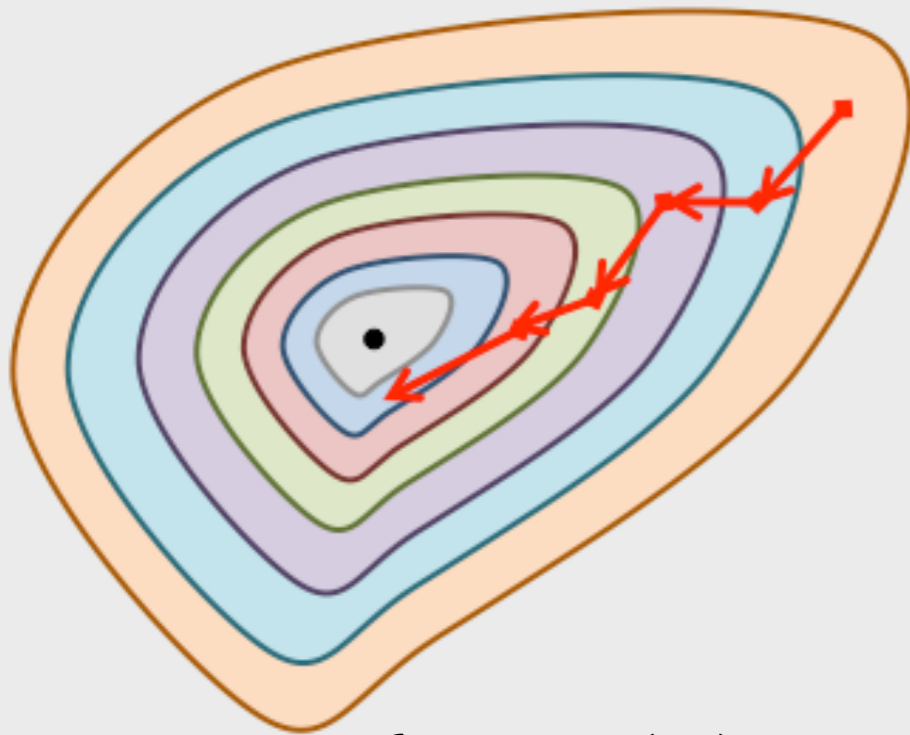


Alice's planning stack



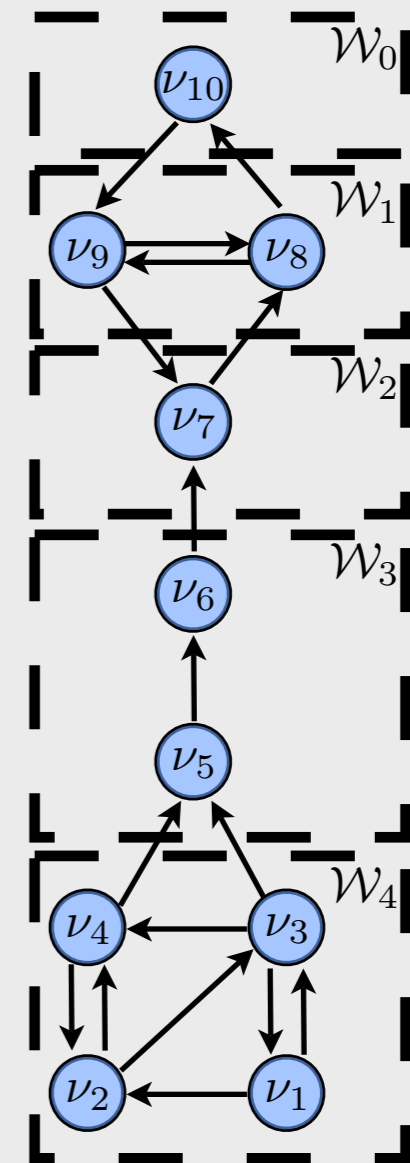
# Contraction Constraints ~ Partial Order

Receding horizon  
control



Level sets  $\{x : V(x) \leq \alpha_i\}$   
induce an order on  $\mathbb{R}^n$ , e.g.,  
 $V$  : control Lyapunov function.

Receding horizon  
temporal logic planning



Norms, level-sets, etc. on continuous spaces do not generalize; but, (partial) orders do!

# Outline

- Setup
- Receding horizon temporal logic synthesis
- Vehicle management systems
- Distributed synthesis

# Vehicle Management Systems



Manages a number of avionics functionalities and their power/computation/communication resources.  
Reacts to the changes in the “environment” in real time.

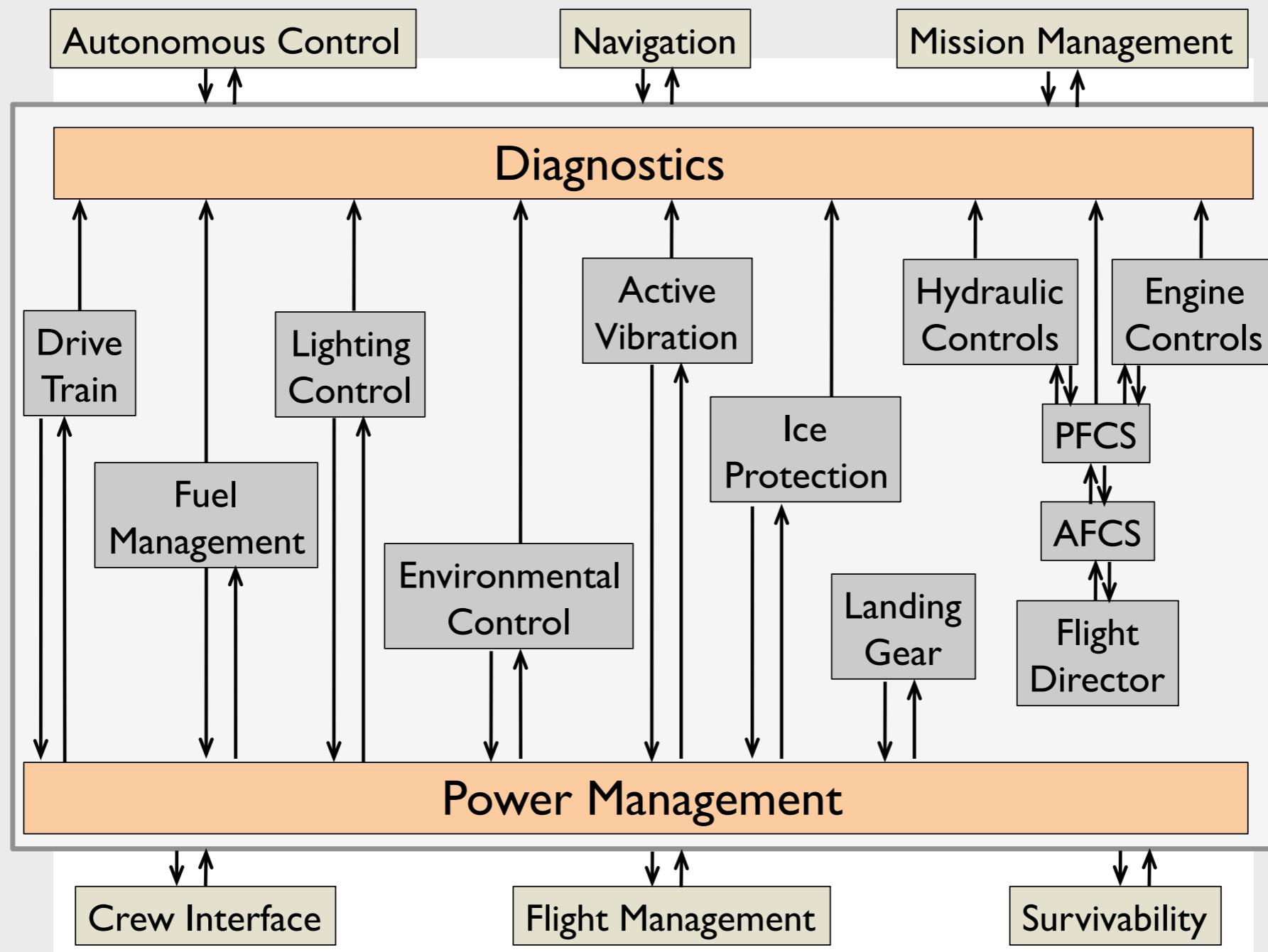
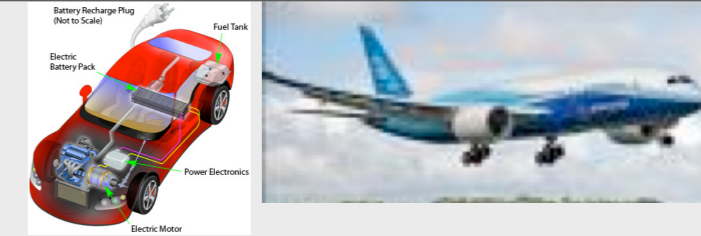
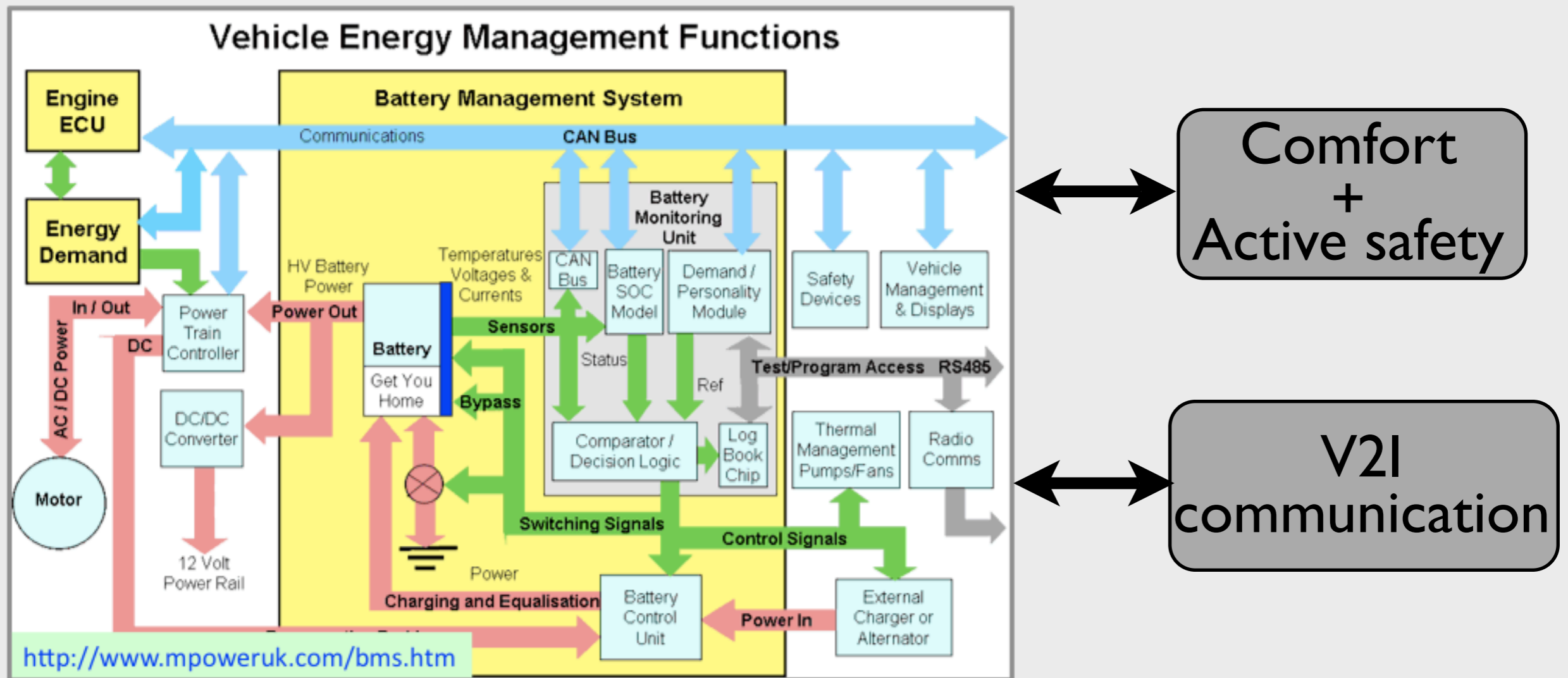


Figure – recreated from a similar figure by W. P. Kinahan, Sikorsky

# Vehicle Management Systems



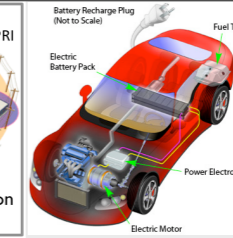
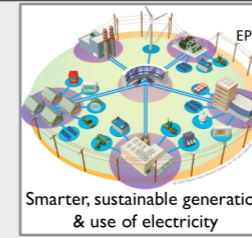
Toyota's Vehicle Dynamics Integrated Management System integrates active safety, comfort, and entertainment functionalities. (pressroom.toyota.com)





# Vehicle Management Systems

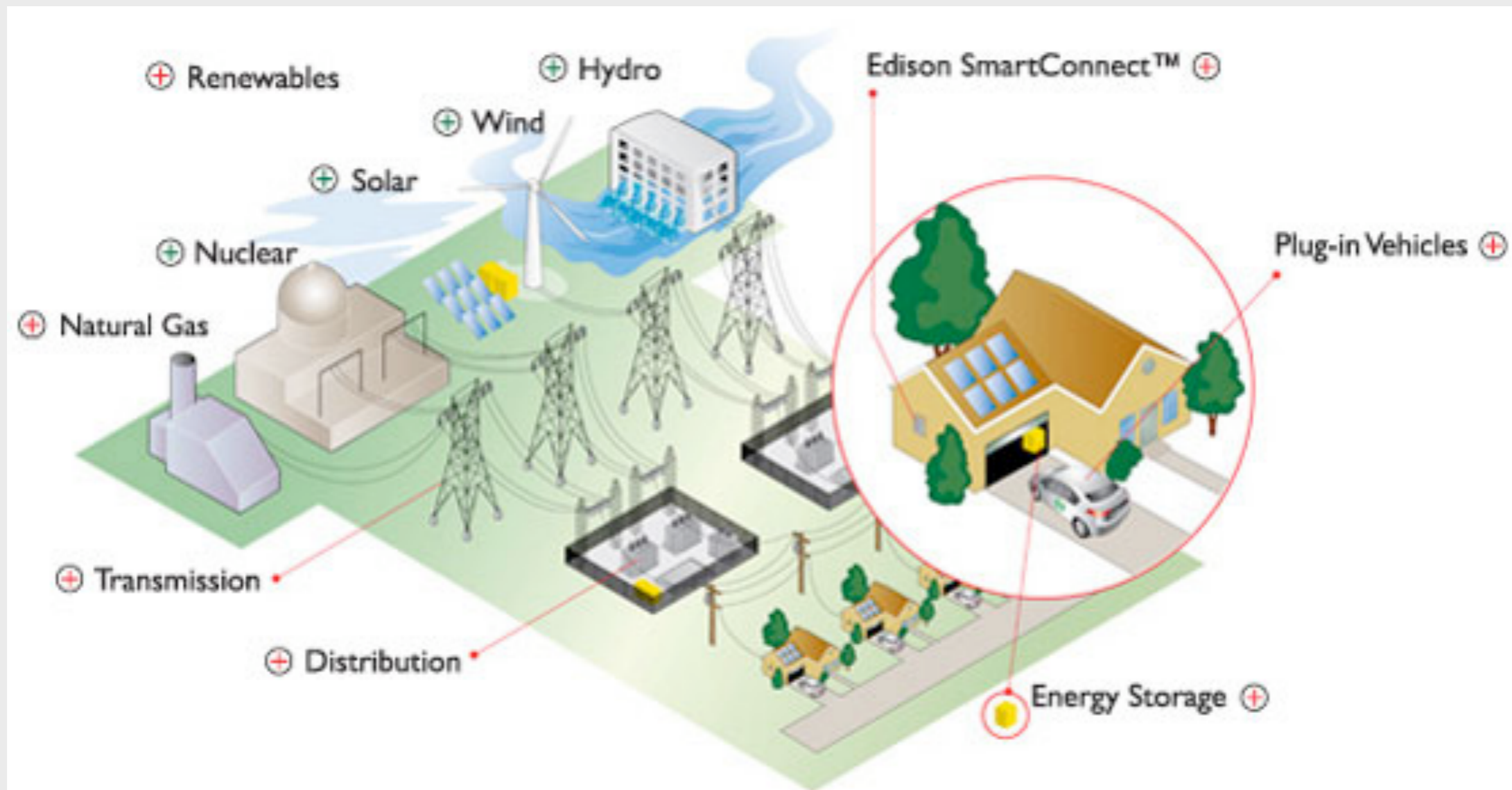
## Energy



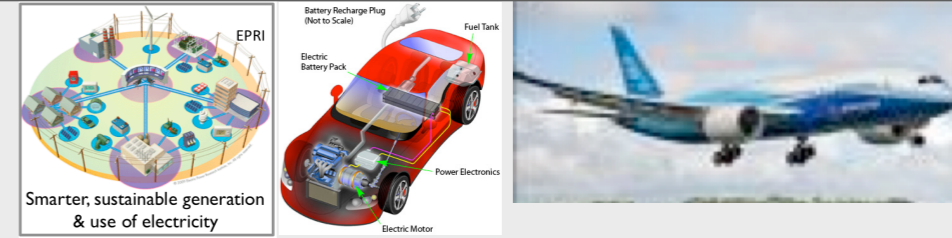
The landscape is changing:

(at multiple levels: devices, buildings, vehicles, power grid)

AMI, energy-smart appliances, electric vehicles, demand response, distributed generation & storage, inverters, AVVC



# ~~Vehicle~~ Management Systems Energy



## Common issues:

- Heterogeneity
  - subsystems
  - requirements
  - (safety) criticality
- Uncertainties of multiple, overlapping scales
- Highly distributed architectures
- Verification of safety & performance
- Managing complexity

## Common driver:

Energy/resource optimization

## Common enabler:

System-level management  
using information systems

# Federated → Integrated Modular

Driver: Support for a number of trends, e.g., more-electric, autonomy,...

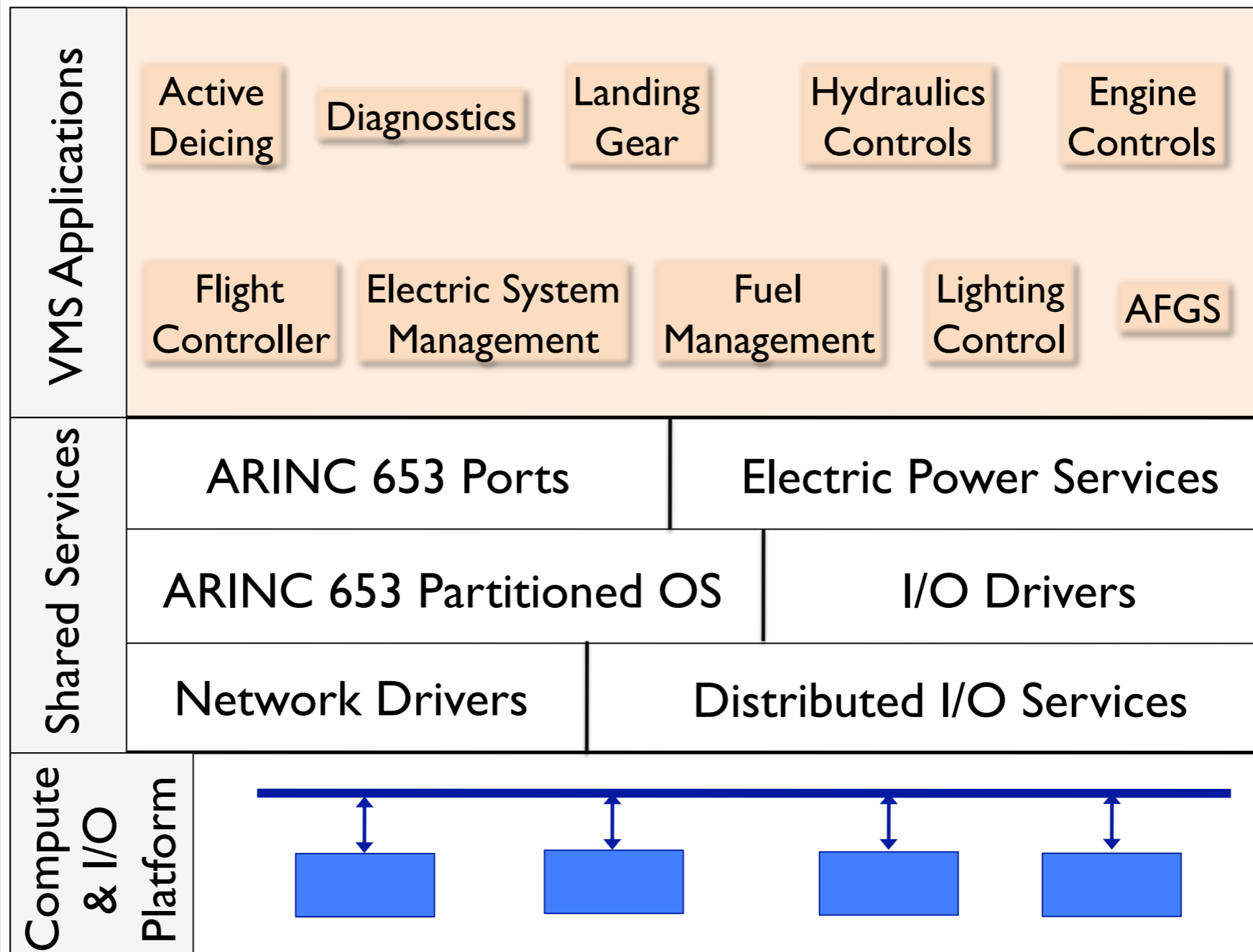


Figure – regenerated from a similar figure by W. P. Kinahan, Sikorsky Aircraft

# Federated → Integrated Modular

Driver: Support for a number of trends, e.g., more-electric, autonomy,...

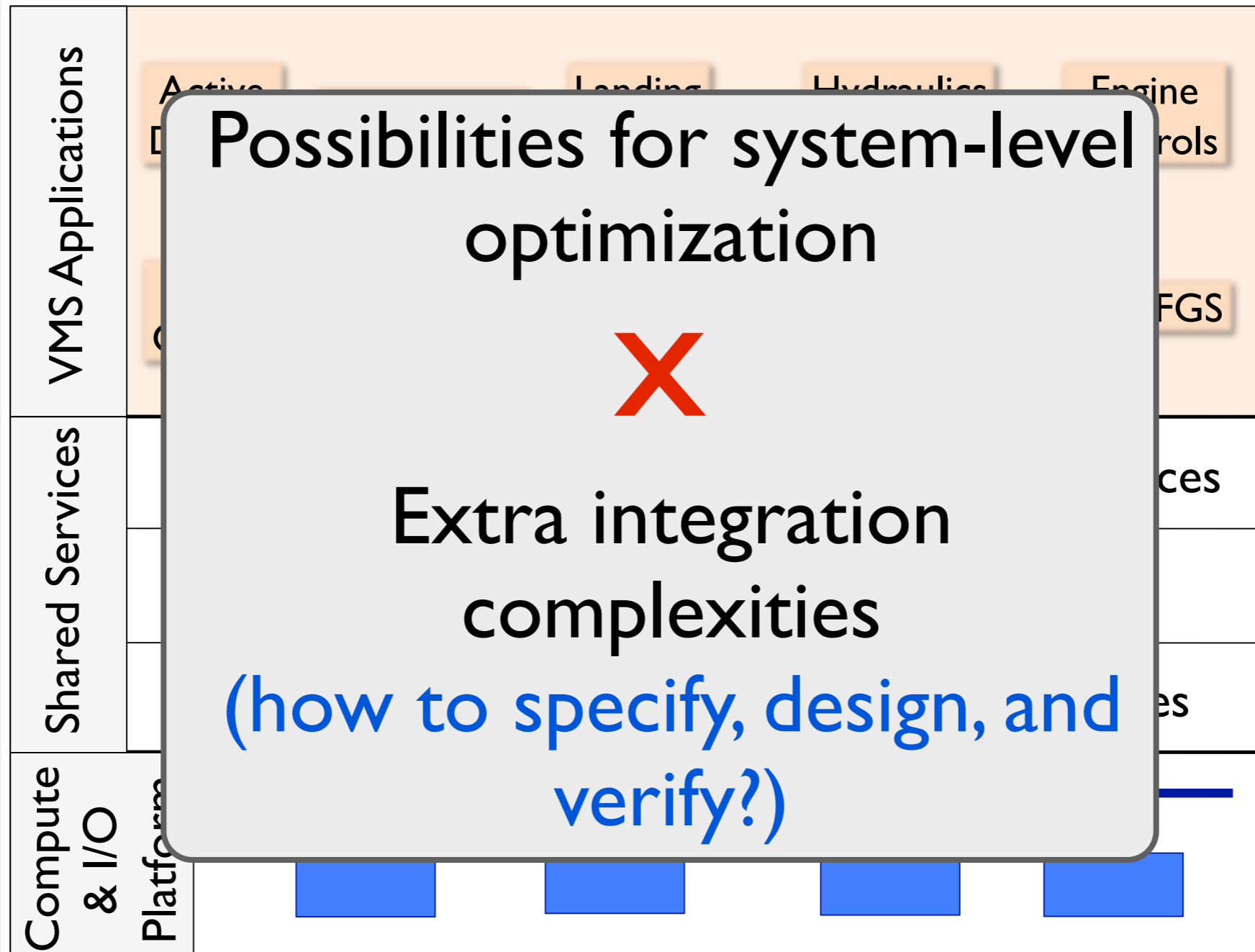


Figure – regenerated from a similar figure by W. P. Kinahan, Sikorsky Aircraft

# Case Study: Control Protocols for VMS

Power management between

- flight controllers
- active de-icing
- environmental control

↑ increasing flight criticality

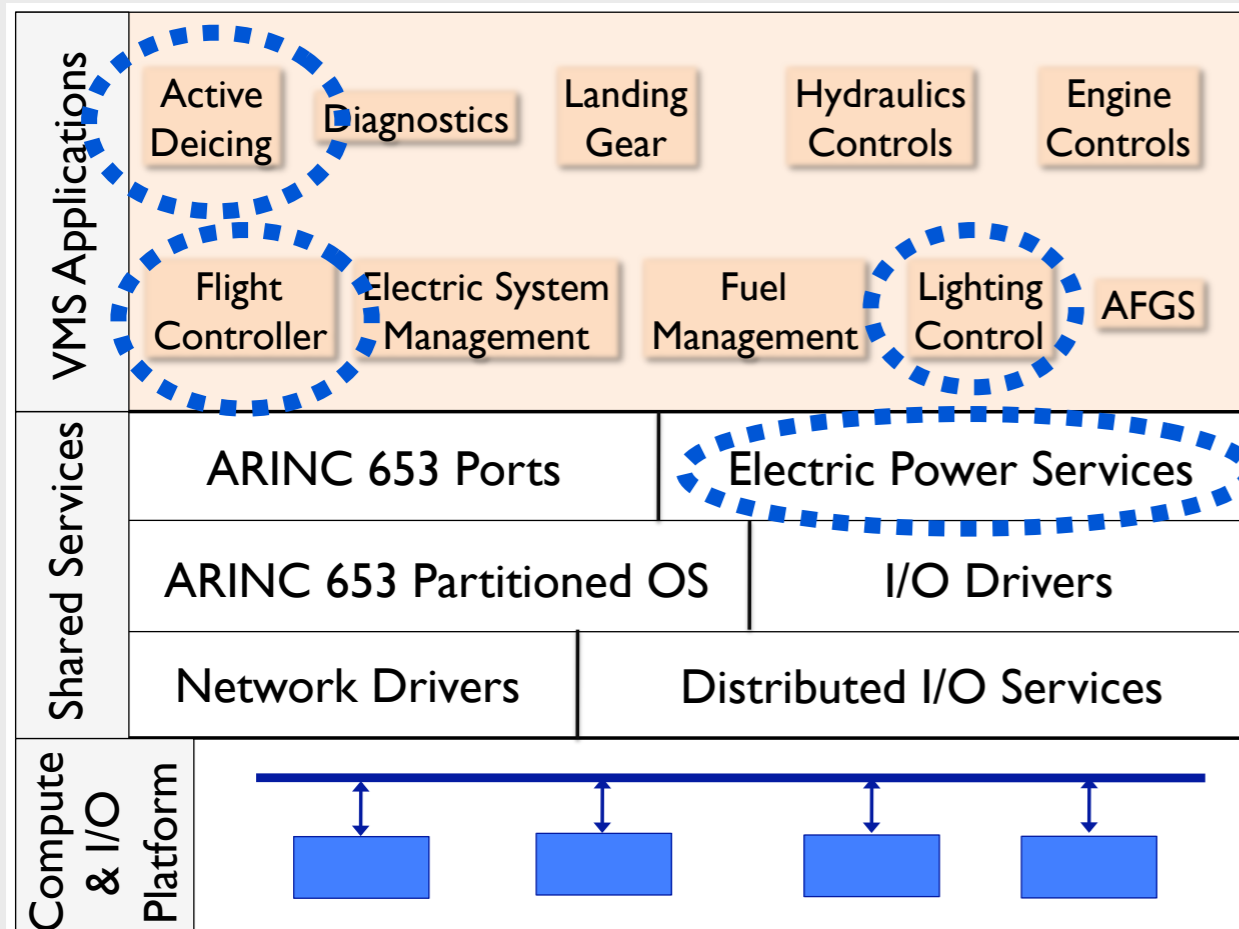


Figure – regenerated from a similar figure by W. P. Kinahan, Sikorsky Aircraft

Environment variables:

wind gust & outside temperature

Controlled variables:

altitude, power supply to different components

Dependent (state) variables:

ice accumulation, energy storage, cabin pressurization



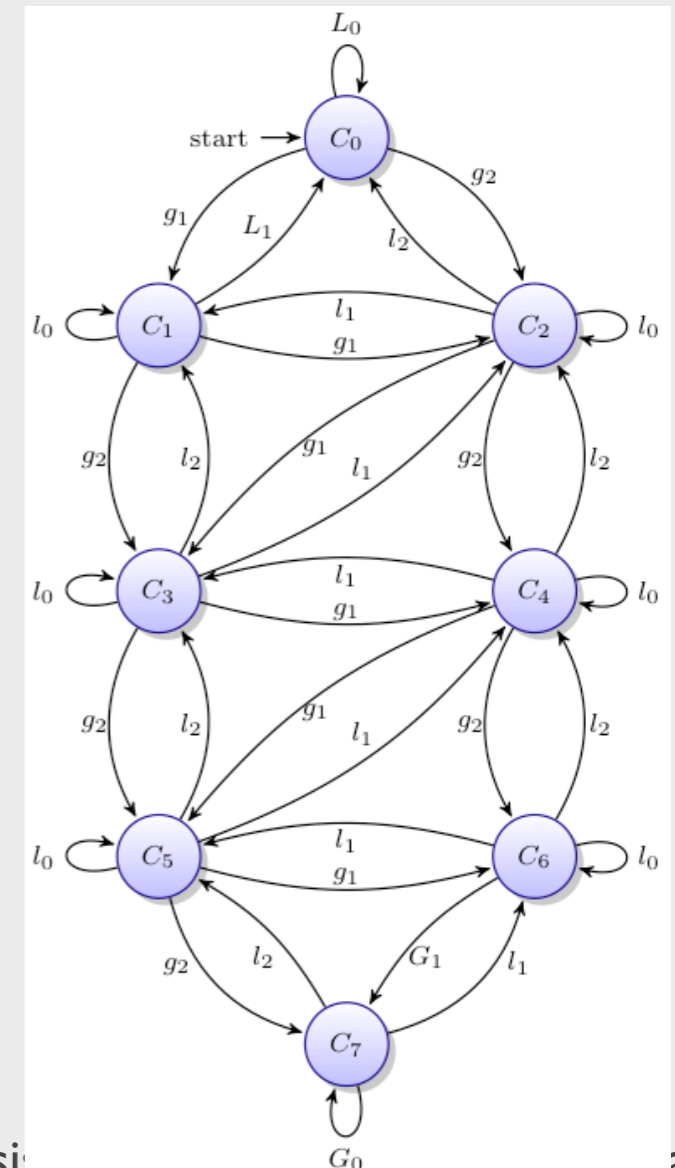
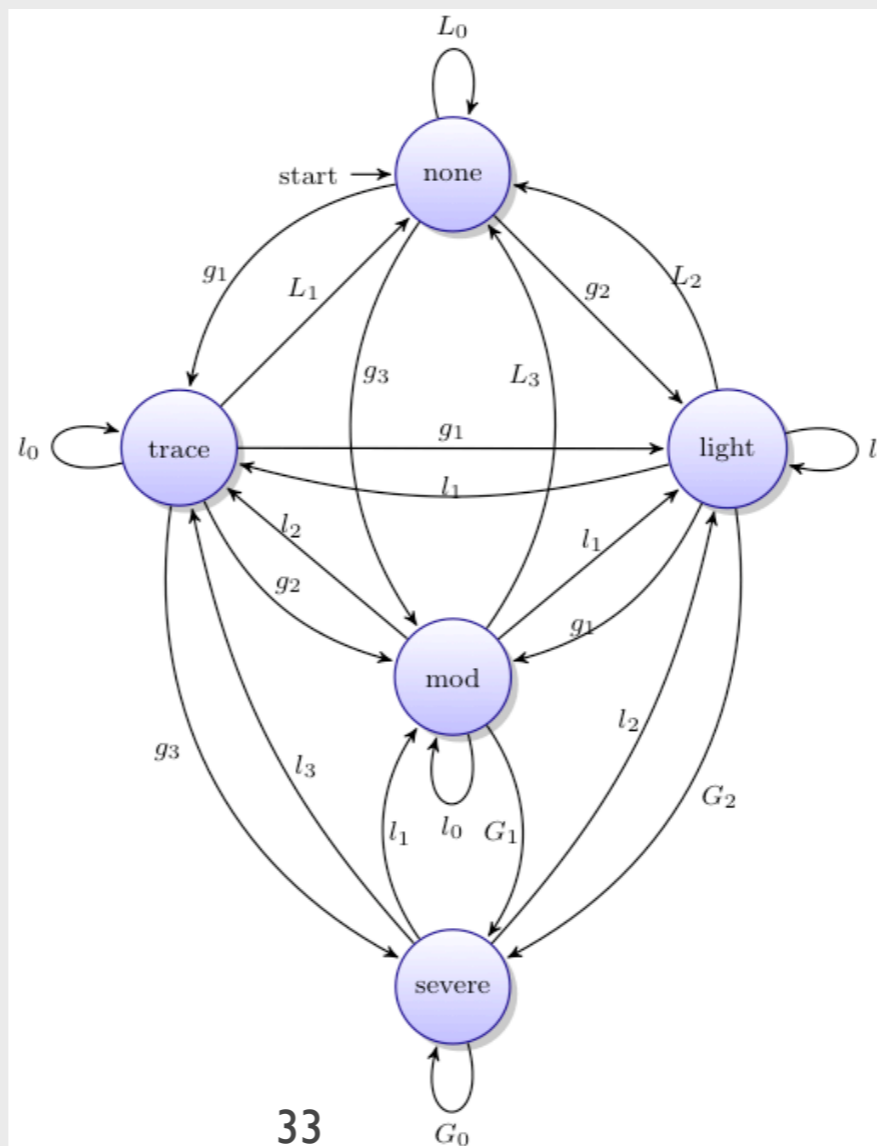
# Specifications

- Limited resources (electric power)
- Safety: prioritization based on flight-criticality & constraint on altitude change and ice accumulation
- Performance: maintain cabin pressure & altitude in desirable ranges
- Environment assumptions on wind gust & temperature

## System model

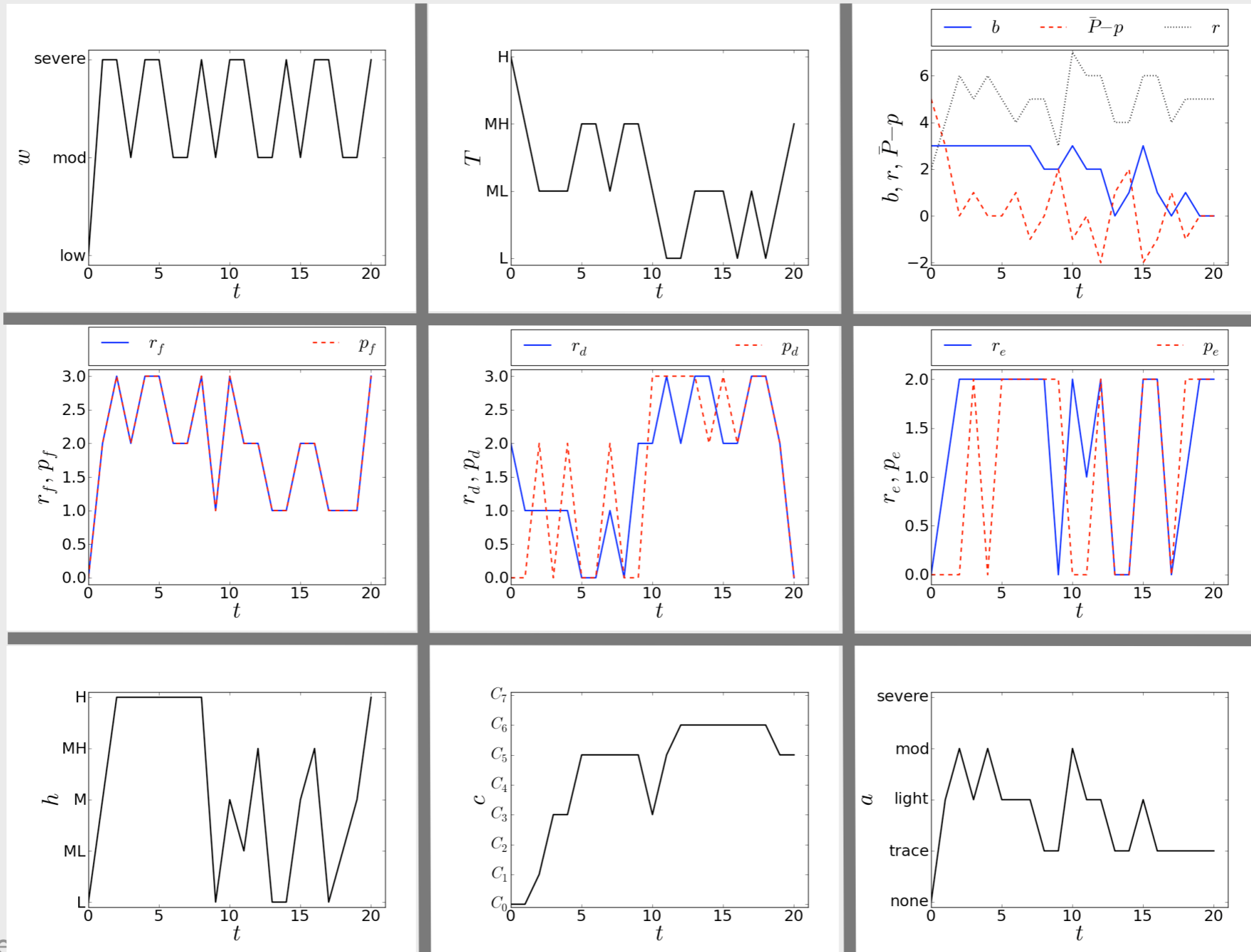
Finite state automata for the evolution of

- ice accumulation
- cabin pressure
- energy storage



# Preliminary results:

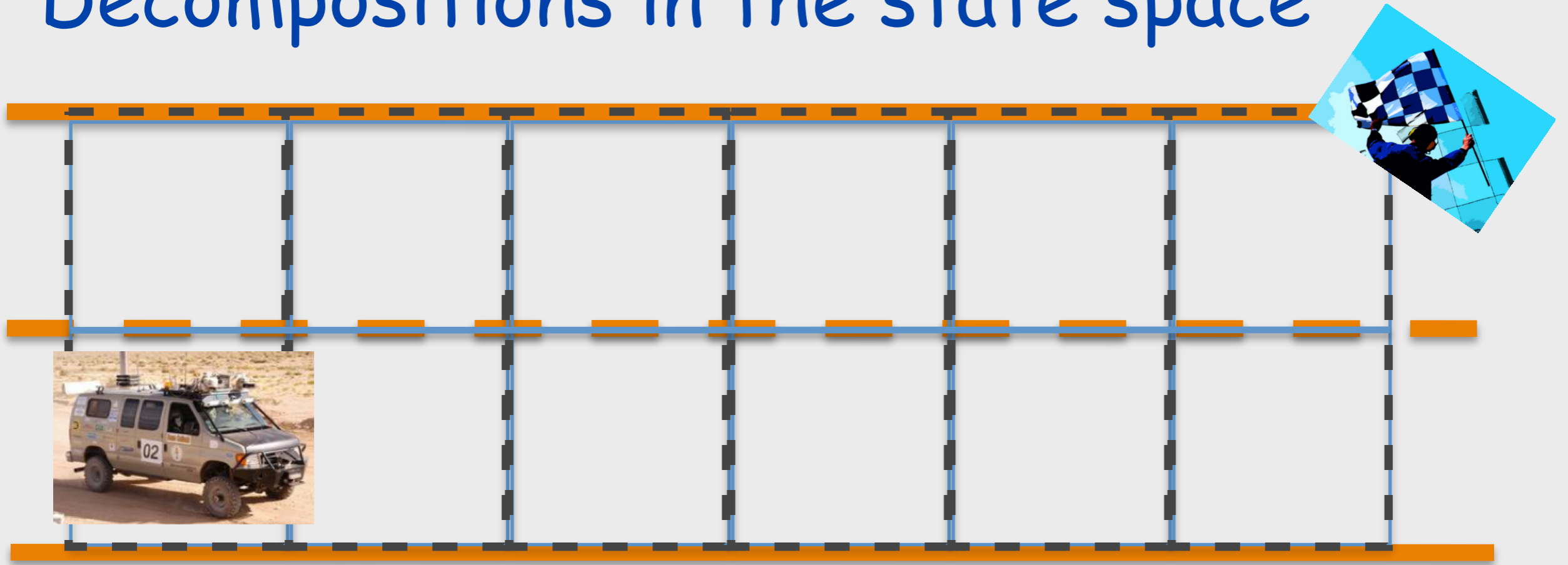
Dynamic power allocation allows reductions in peak power (i.e., generator weight) requirements.



# A sample of open issues

- Optimality vs. feasibility
- Hard time constraints
- Design-for-verification
- Incremental synthesis/verification
- Scalability by exploiting the underlying structure

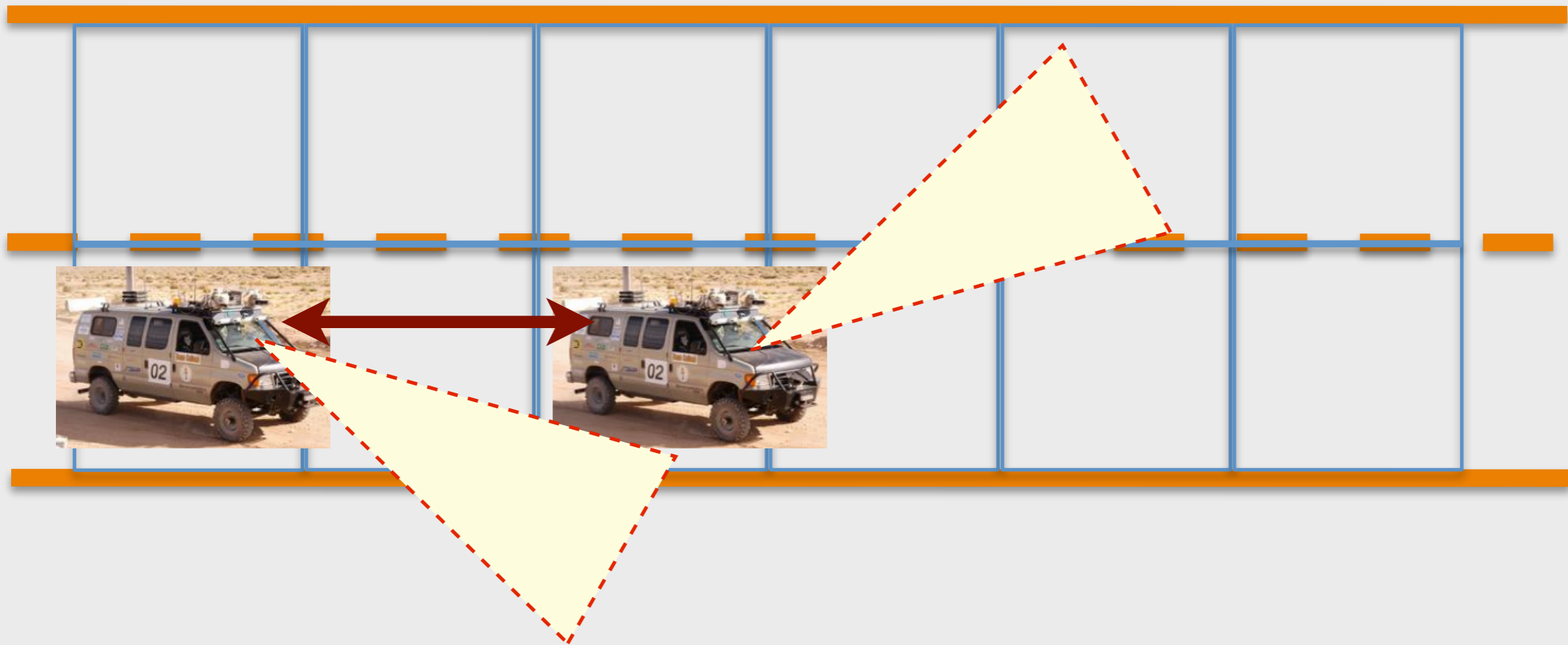
# Decompositions in the state space



Decompositions  
induced by ...

receding horizon	goal
distributed synthesis	underlying network

# Decompositions in the state space

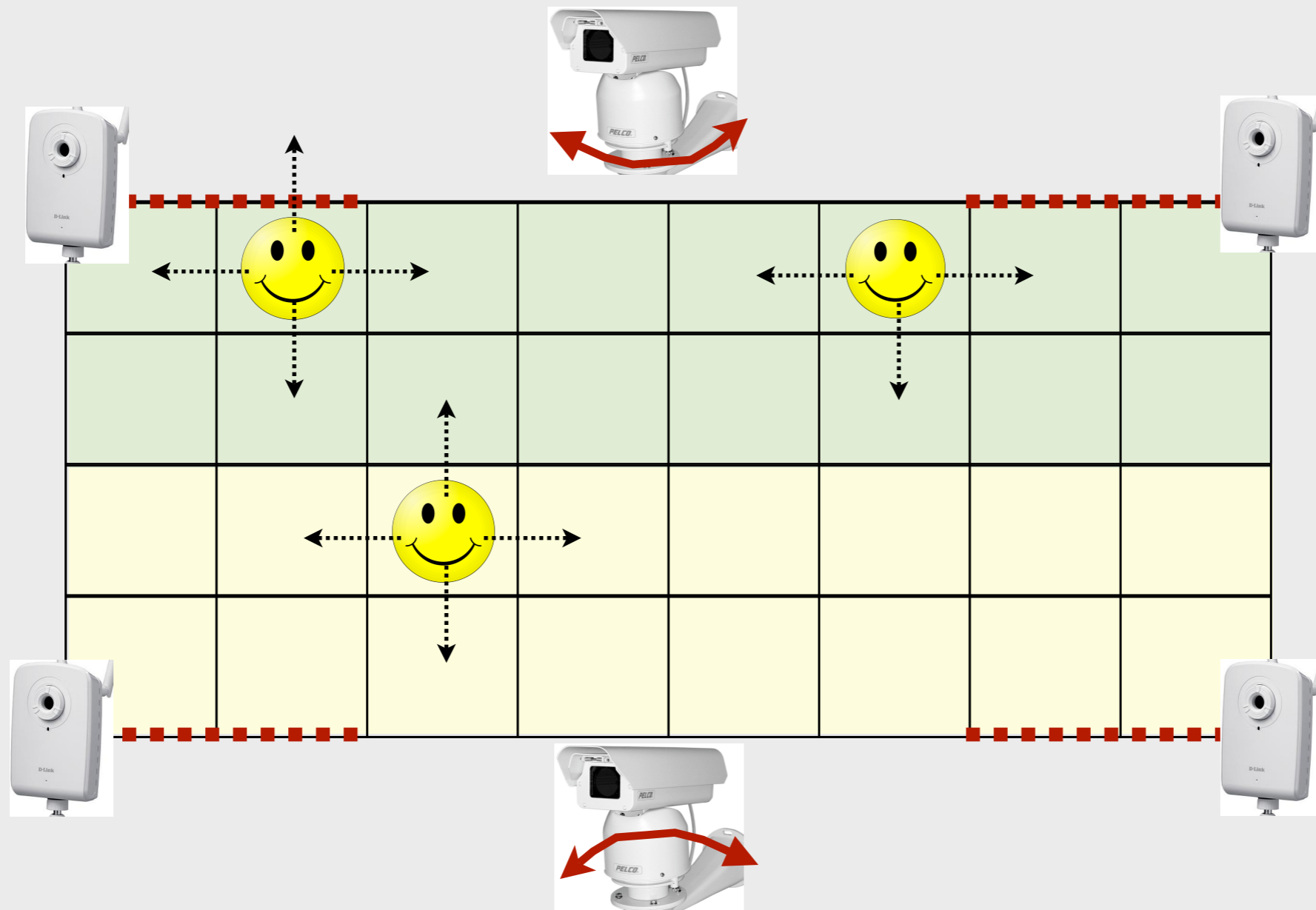


Decompositions  
induced by ...

receding horizon	goal
distributed synthesis	underlying network

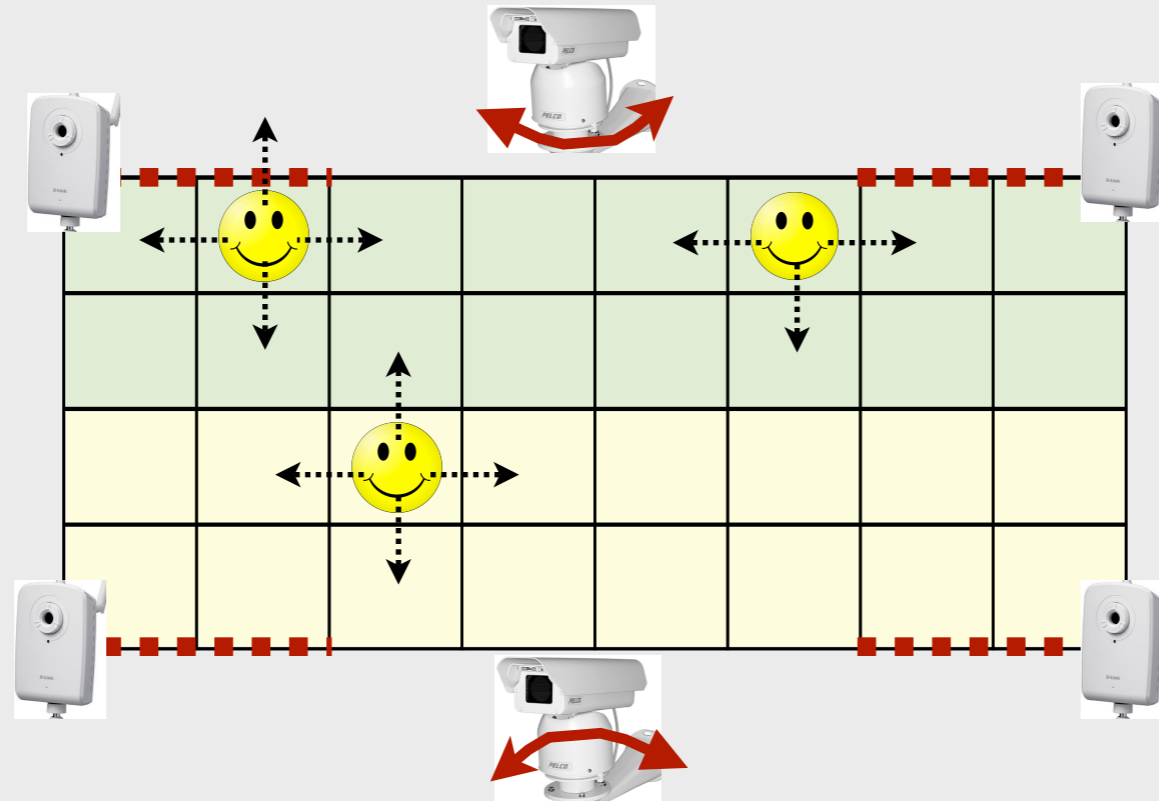


Smart camera networks {  
- static cameras for tracking targets  
- pan-tilt-zoom (PTZ) for active recognition



**Goal:** synthesize control protocols for PTZ to ensure that one high resolution image of each target is captured at least once

# Synthesis of protocols for active surveillance



## System:

- region of view of PTZs
- governed by finite state automata

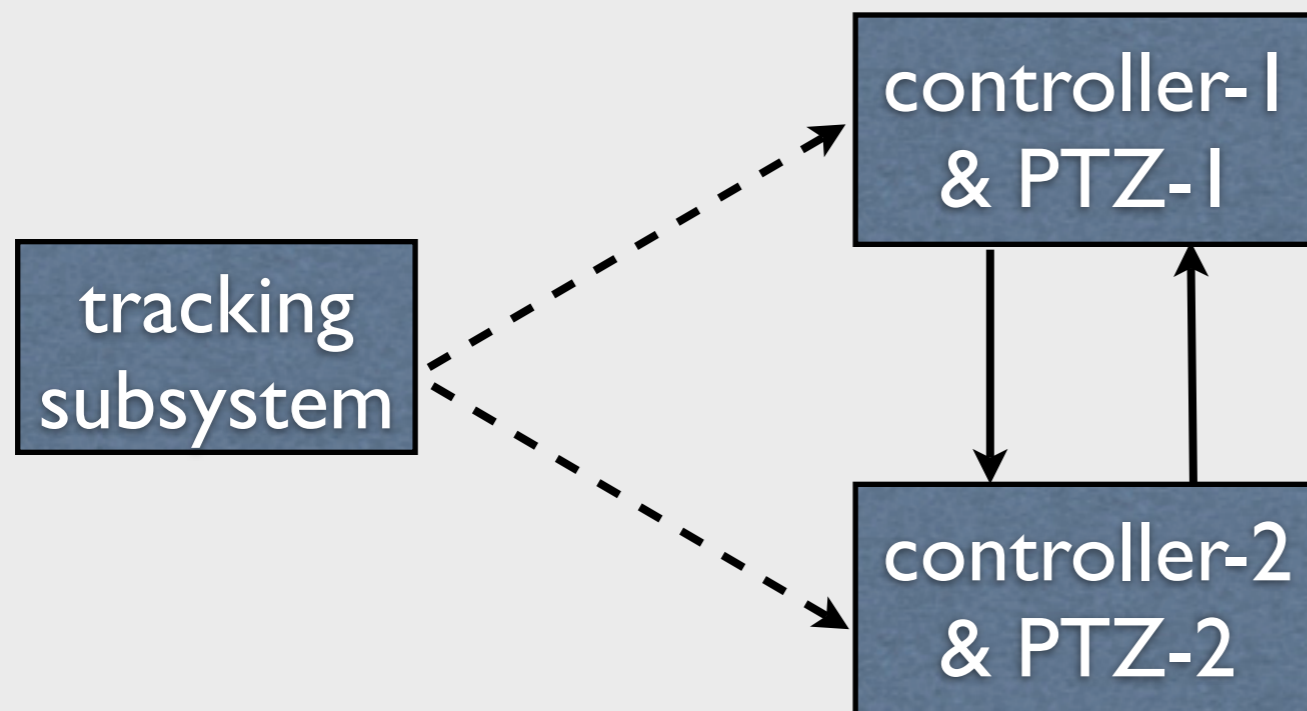
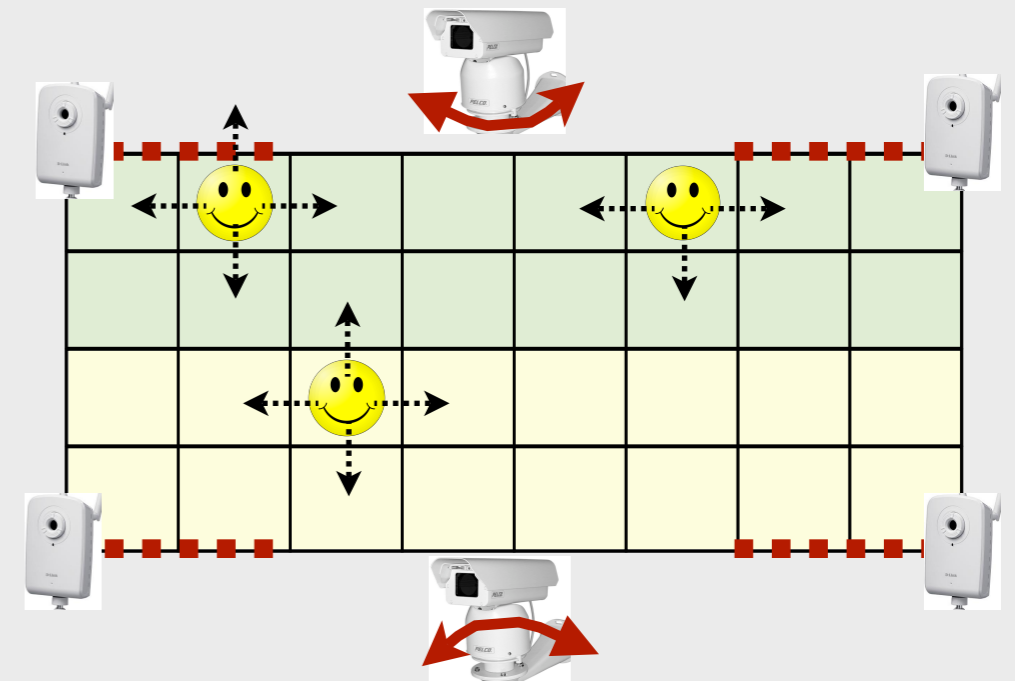
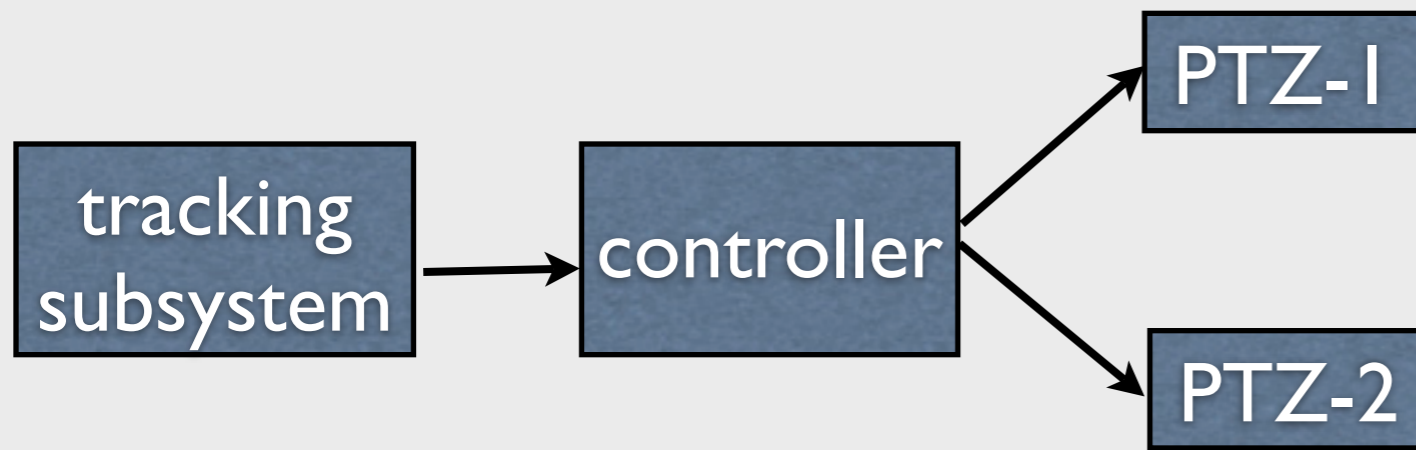
## Additional requirement:

- Zoom-in the corner cells infinitely often.

## Environment specifications:

- At most  $N$  targets at a time.
- Every target remains at least  $T$  time steps and eventually leaves.
- Can only enter/exit through doors.
- Can only move to neighbors.

# Centralized vs. decentralized control architecture



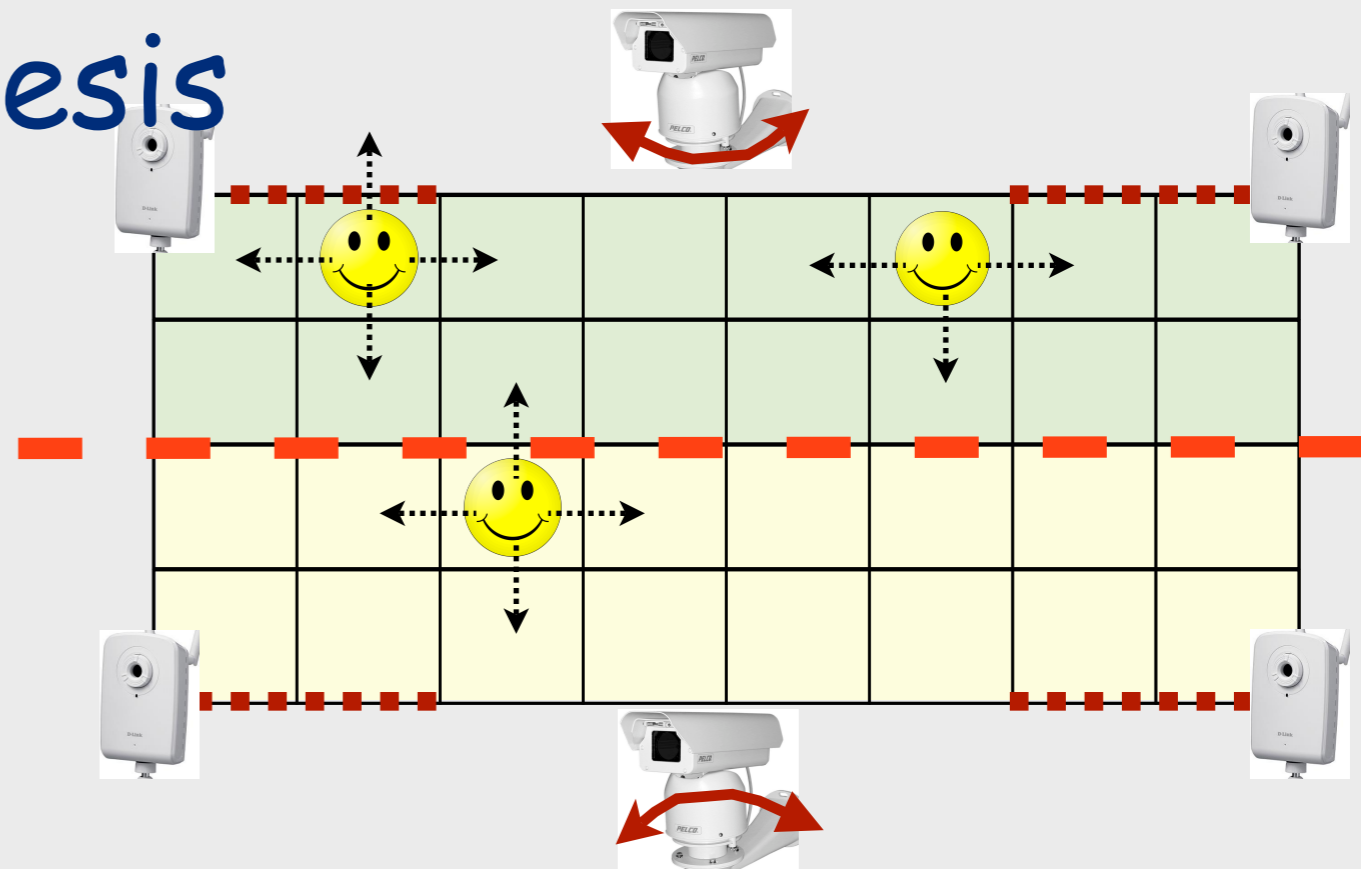
How to design control protocols that can be

- synthesized
- implemented in a decentralized way?

What information exchange & interface models are needed?

# Compositional Synthesis

Goal: Find control protocols for PTZ-1 & PTZ-2 so that  $\varphi_e \rightarrow \varphi_s$  holds.



Simple & not very useful composition:

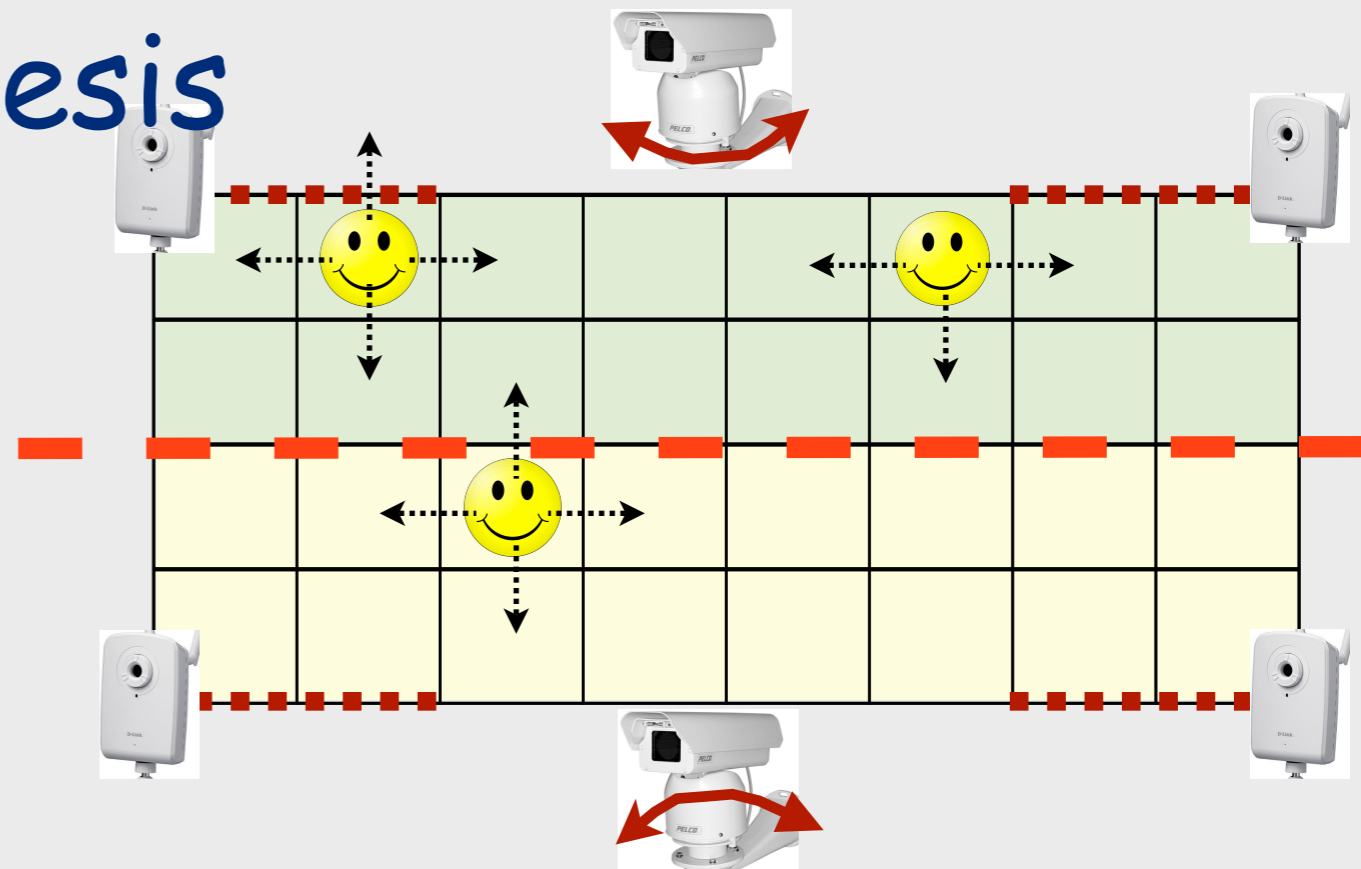
Any execution of the env't, satisfying  $\varphi_e$ , also satisfies  $\varphi_{e_1} \wedge \varphi_{e_2}$

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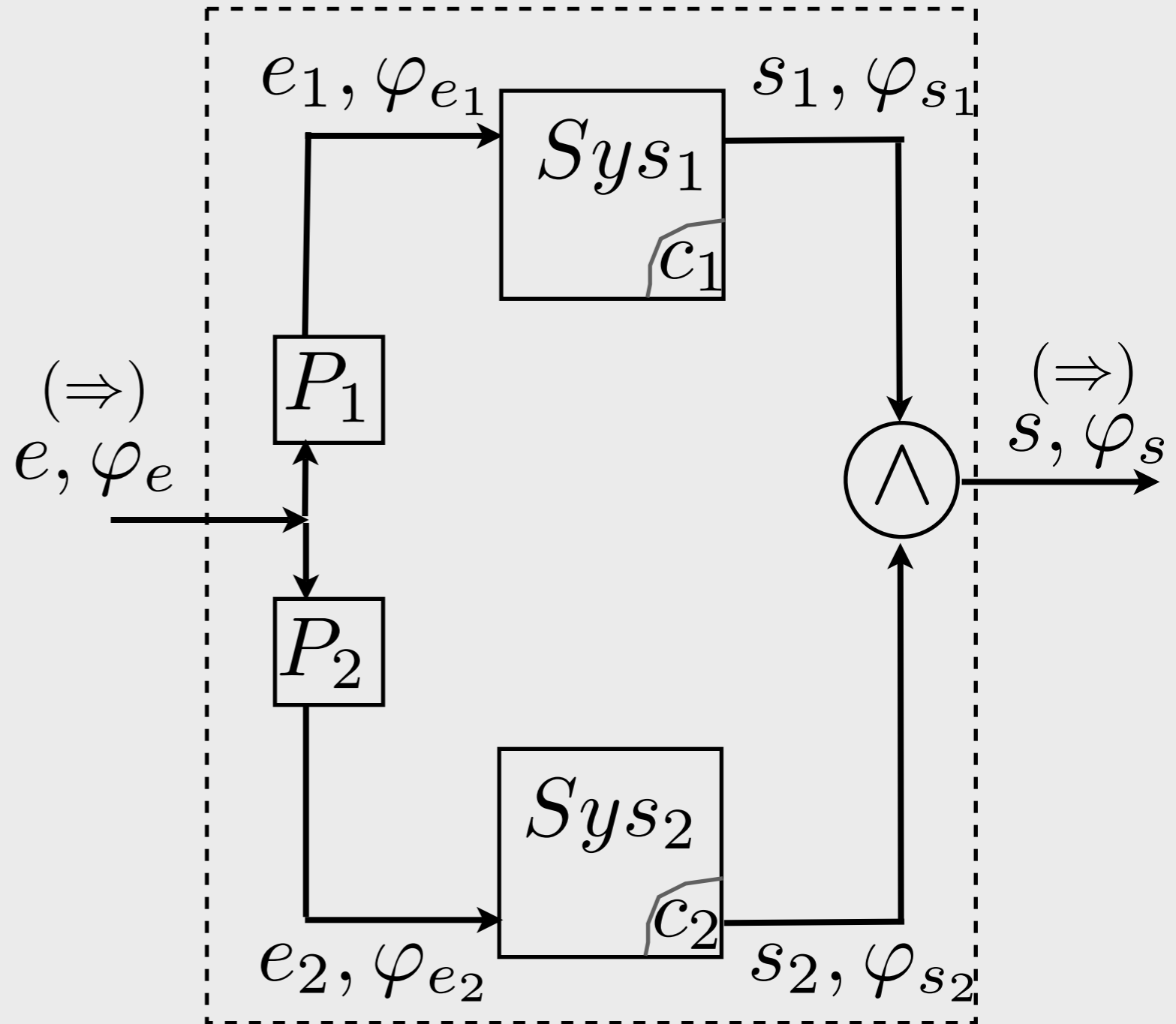
➔  $\varphi_e \rightarrow \varphi_s$  is realized.



# Central



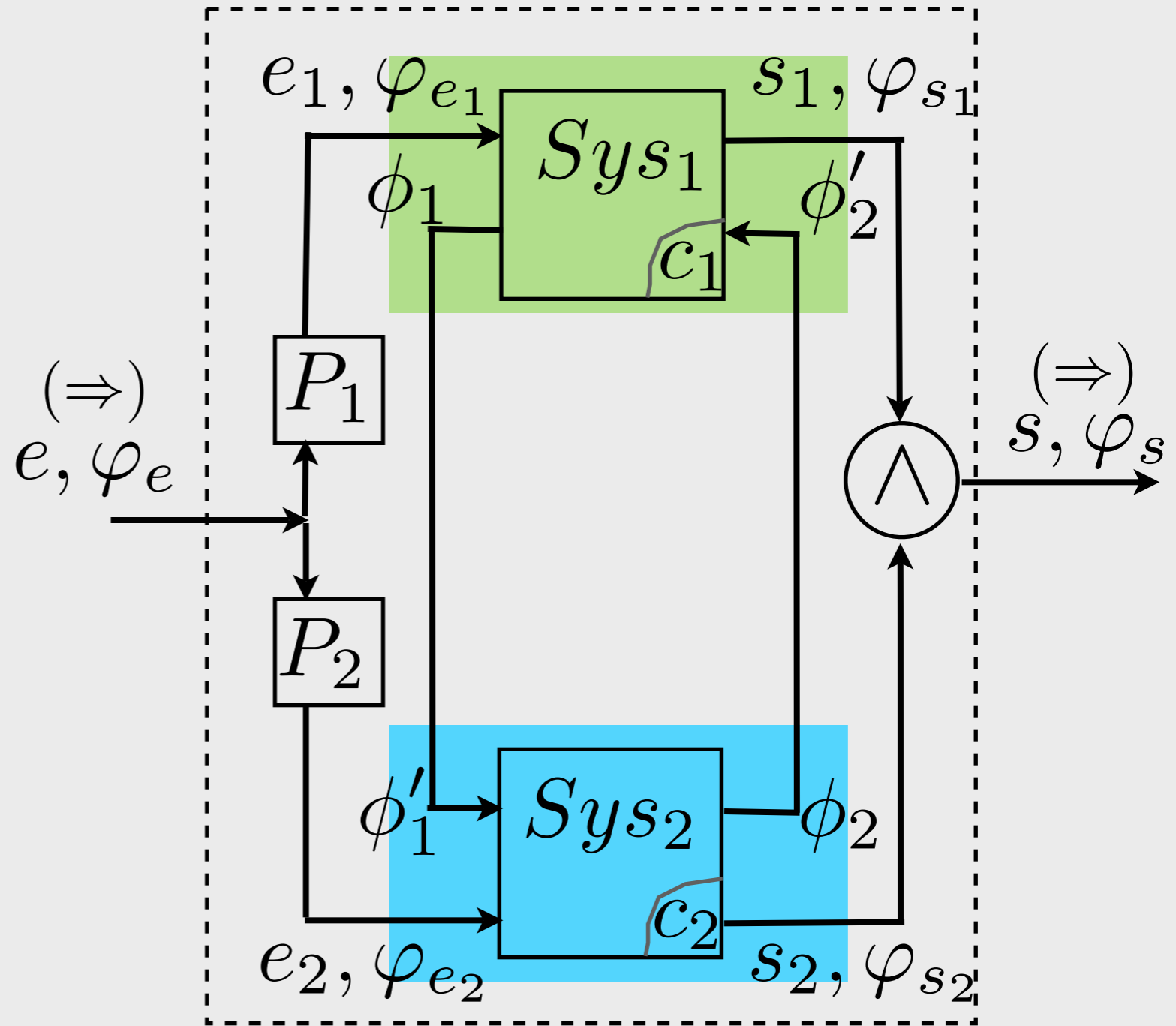
# Compositional



# Central



# Compositional



# (Refined) Compositional Synthesis

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OTWM@ICCPSS I (s)

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Refined interfaces:

There exist control protocols that realize

$$(\phi'_2 \wedge \varphi_{e_1}) \rightarrow (\varphi_{s_1} \wedge \phi_1) \quad \& \quad (\phi'_1 \wedge \varphi_{e_2}) \rightarrow (\varphi_{s_2} \wedge \phi_2)$$

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# (Refined) Compositional Synthesis

## As before:

Any execution of the env't, satisfying  $\varphi_e$ , also satisfies  $\varphi_{e_1} \wedge \varphi_{e_2}$

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## Refined interfaces:

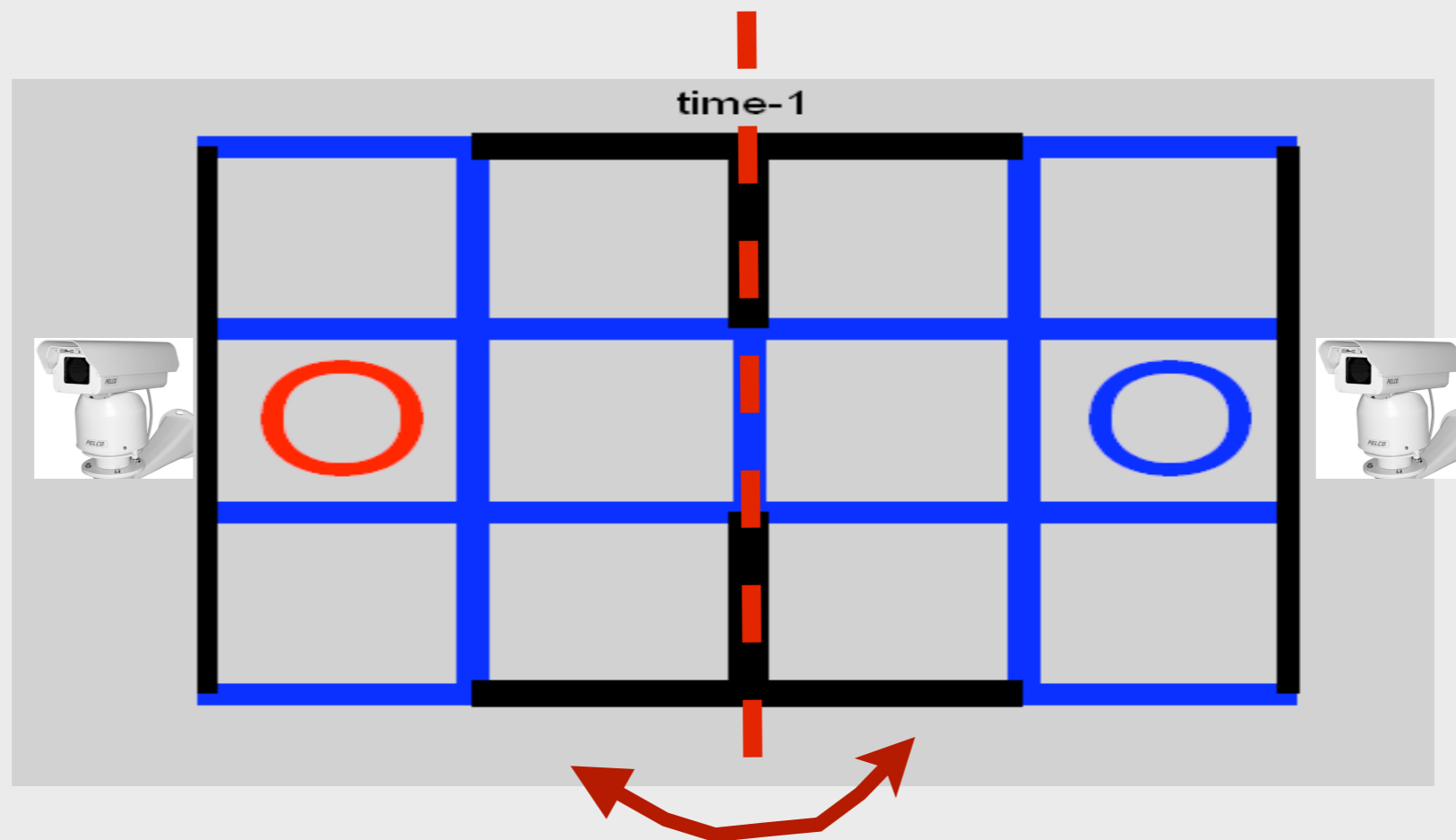
There exist control protocols that realize  
 $(\phi'_2 \wedge \varphi_{e_1}) \rightarrow (\varphi_{s_1} \wedge \phi_1)$    &    $(\phi'_1 \wedge \varphi_{e_2}) \rightarrow (\varphi_{s_2} \wedge \phi_2)$

## For soundness and to avoid circularity:

$\square (\phi_i \rightarrow \circ\phi'_i)$    for  $i = 1, 2$

  $\varphi_e \rightarrow \varphi_s$  is realized.

# Application to a (very simple) smart camera network



IsZoomed & StepsInZone

$\phi_1$  and  $\phi'_1$   
limit the number of unzoomed targets  
entering zone 2 from zone 1

# Summary

Receding horizon temporal logic synthesis

Distributed synthesis

Applications

- Vehicle management systems
- Autonomous driving
- Active surveillance

## A sample of open issues

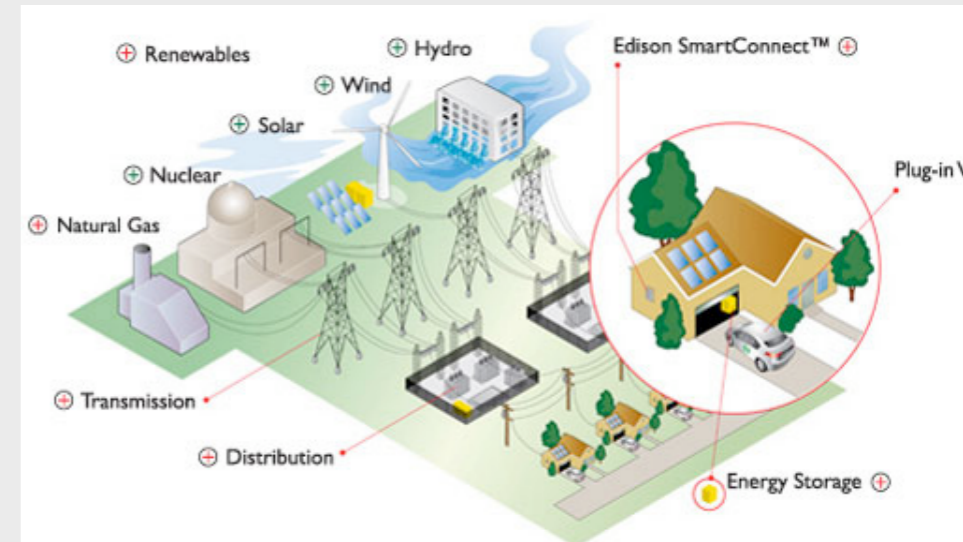
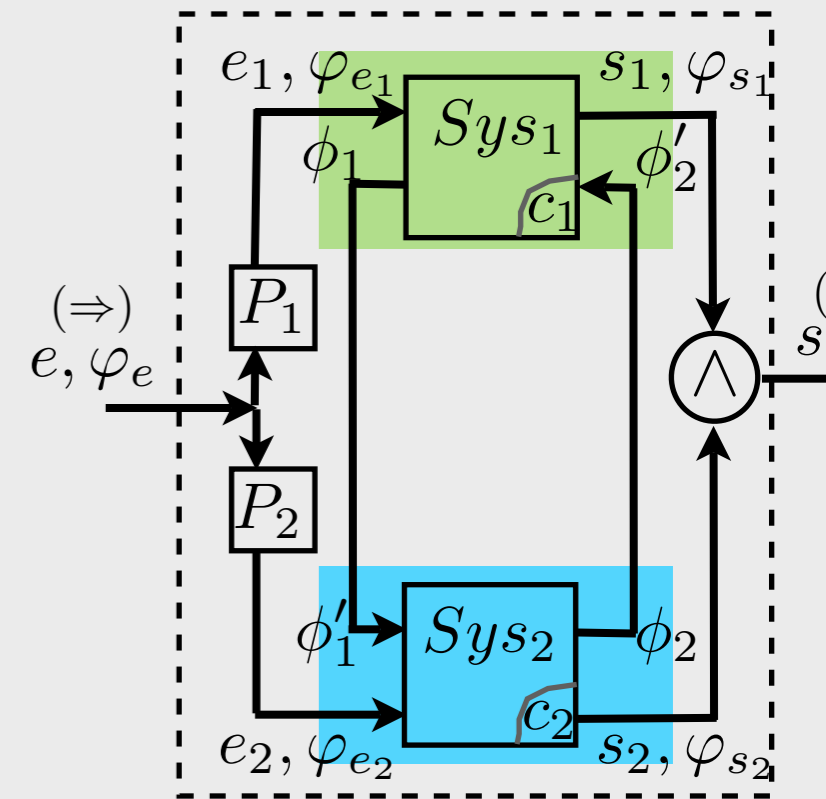
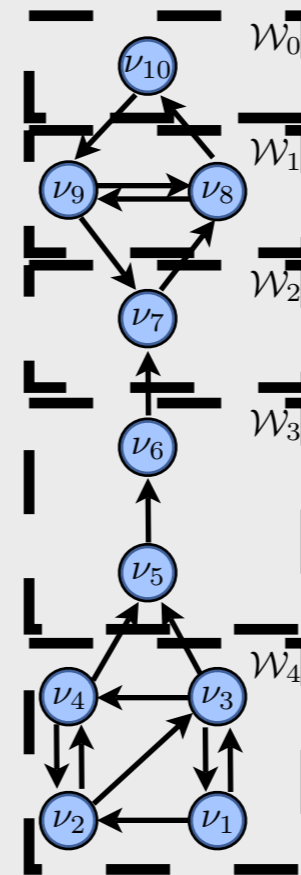
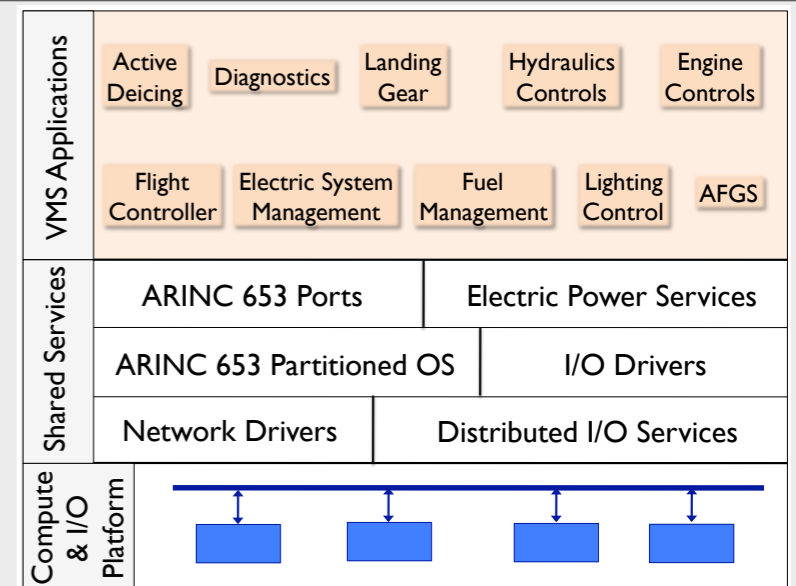
Optimality vs. feasibility

Hard time constraints

Incremental synthesis/verification

Fidelity of models/abstractions

Exploiting the underlying structure



# All references

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available at

[www.cds.caltech.edu/~UTopcu](http://www.cds.caltech.edu/~UTopcu)