Local proof transformations for flexible interpolation and proof reduction

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Outline

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Background

Formal Verification in Lugano, Switzerland

- Program Verification

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  - Model checking code (LoopFrog, Synergy, SatAbs (with Oxford), FunFrog), ANSI-C
  - Efficient decision procedures as computational engines of verification (OpenSMT)

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- Abstractions
  - Program Summarization [ATVA'08], [ASE'09]
    - Avoids fix-point computation by constructing symbolic abstract transformers instead
    - Performs sound over-approximation of (unbounded) loops
    - Precision is tuned by selection of abstract domains
    - Exploits efficiency of SAT/SMT solvers

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Formal Verification in Lugano, Switzerland

- Program Termination [CAV'10, TACAS'11]
  - Integration of Loop Summarization with Termination Analysis
  - Compositional Transition Invariants avoid all paths computation of termination checks
  - Simple abstract domains are used for termination checks

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  - Integration of Loop Summarization with Termination Analysis
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  - Simple abstract domains are used for termination checks
- Synergy of Abstractions [STTT'10]
  - Interleaves precise and over-approximated abstractions
  - Reduces CEGAR iterations
  - Removes multiple counterexamples within a single refinement step
  - Localizes precise abstraction/refinement to relevant parts of the program

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Formal Verification in Lugano, Switzerland

- Model checking mobile code [IFM'08], [JFAC'10]
  - Specification language for security policies
  - Formalization of mobile code distribution net
  - Location-specific abstractions and model checking of security policies

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- Boolean and Theory Reasoning (SMT)
  - Procedure for bit-vector extraction and concatenation [ICCAD'09]
    - Reduces formulae to the theory of equality to avoid, when possible, expensive reduction to SAT

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  - Generation of explanations in theory propagation [MEMOCODE'10]
    - Computes explanations on demand by reusing the consistency check algorithm for a generic theory T.

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Formal Verification in Lugano, Switzerland

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  - Generation of interpolants (for QF EUF, RDL)
  - Proof manipulation for interpolation [ICCAD'10]
  - Proof reduction [HVC'10]

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  - Generation of interpolants (for QF EUF, RDL)
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  - Proof reduction [HVC'10]
  - Solver, *OpenSMT*, combines MiniSAT2 SAT-Solver with state-of-the-art decision procedures for QF EUF, LRA, BV, RDL, IDL
    - *Extensible*: the SAT-to-theory interface facilites design and plug-in of new decision procedures
    - *Incremental*: suitable for incremental verification
    - *Open-source*: available under GPL license
    - *Efficient*: currently the fastest open-source SMT Solver for QF UF, IDL, RDL, LRA according to SMT-Comp'10.

- Boolean and Theory Reasoning (SMT)
  - Generation of interpolants (for QF EUF, RDL)
  - Proof manipulation for interpolation [S.F. Rollini, R. Bruttomesso, N. Sharygina, A. Tsitovich, ICCAD'10]
  - Resolution proof reduction [S.F. Rollini, R. Bruttomesso, N. Sharygina, HVC'10]

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1. Background
2. Motivation and Related Work
3. Contribution
   - Proof Transformation for Interpolation and Reduction
4. Summary and Future Work

- Resolution proofs find application in several ambits

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    - Interpolation-based model checking
    - Abstraction techniques
    - Unsatisfiable core extraction in SAT/SMT
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  - Clean structure of proofs is required for interpolation generation

Proof Transformation and Reduction

Motivation

- Resolution proofs find application in several ambits
  - Interpolation-based model checking
  - Abstraction techniques
  - Unsatisfiable core extraction in SAT/SMT
  - Automatic theorem proving
- Problems
  - Clean structure of proofs is required for interpolation generation
  - Size affects efficiency
  - Size can be exponential w.r.t. input formula

Notation

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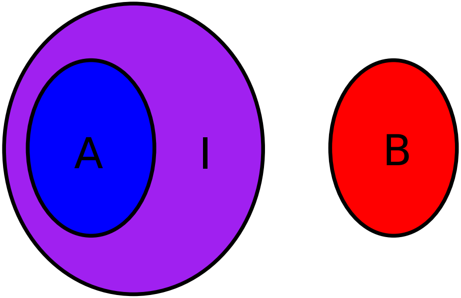

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  - Problem solving by means of resolution based engines (SAT solvers, SMT solvers)

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  - Example

    $A \triangleq (5x - y \leq 1) \wedge (y - 5x \leq -1) \qquad B \triangleq (y - 5z \leq 3) \wedge (5z - y \leq -2)$

SAT and SMT

Proofs and Solving Engines

- $A \wedge B$ unsatisfiable: certificate of unsatisfiability

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- SMT solver
  - DPLL SAT solver
  - Theory solver

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- State-of-the-art approach [Pudlák97, McMillan04]

Interpolation
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  - Computation of I from proof structure in linear time

Resolution System

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- Literal p $\overline{p}$

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- Clause $p \vee \overline{q} \vee r \vee \ldots \rightarrow p\overline{q}r\ldots$ Empty clause $\bot$

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- Resolution rule $$\frac{pC \qquad \overline{p}D}{CD}\, p$$

Antecedents: pC $\overline{p}D$ Resolvent: CD Pivot: p

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  Antecedents: $pC\ \overline{p}D$ Resolvent: CD Pivot: p

- Resolution proof of unsatisfiability of a set of clauses S
  - Tree
  - Leaves as clauses of S
  - Intermediate nodes as resolvents
  - Root as unique empty clause

Resolution Proofs
SAT

- $A \triangleq \{\overline{p}\overline{q}, p\overline{q}\}$ $B \triangleq \{q\overline{r}, qr\}$

Resolution Proofs

SAT

- $A \triangleq \{\overline{p}\,\overline{q},\ p\overline{q}\}$ $\qquad B \triangleq \{q\overline{r},\ qr\}$
- Proof of unsatisfiability

$$\dfrac{\dfrac{\overline{p}\,\overline{q} \qquad p\overline{q}}{\overline{q}}\ p \qquad\qquad \dfrac{q\overline{r} \qquad qr}{q}\ r}{\bot}\ q$$

Interpolant Generation

SAT [Pudlák97]

- Computation of interpolant I for $A \wedge B$ from proof structure

Interpolant Generation

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Resolution Proofs

SMT

- $A \triangleq \{\overbrace{\boxed{(5x - y \leq 1)}}^{p}, \overbrace{\boxed{(y - 5x \leq -1)}}^{q}\}$ $B \triangleq \{\overbrace{\boxed{(y - 5z \leq 3)}}^{r}, \overbrace{\boxed{(5z - y \leq -2)}}^{s}\}$

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  - LIA: $\overbrace{\boxed{(x - z \leq 0)}}^{t}\overbrace{\boxed{(x - z \geq 1)}}^{u}$
  - LRA: $\overbrace{\boxed{(5x - y \not\leq 1)}}^{\overline{p}}\overbrace{\boxed{(y - 5z \not\leq 3)}}^{\overline{r}}\overbrace{\boxed{(x - z \not\geq 1)}}^{\overline{u}}$

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  - LRA: $\overbrace{(y - 5x \not\leq -1)}^{\overline{q}}\ \overbrace{(5z - y \not\leq -2)}^{\overline{s}}\ \overbrace{(x - z \not\leq 0)}^{\overline{t}}$

Resolution Proofs

SMT

- $A \triangleq \{p, q\}$ $\quad B \triangleq \{r, s\}$ $\quad L \triangleq \{tu, \overline{p}\overline{r}\overline{u}, \overline{q}\overline{s}\overline{t}\}$

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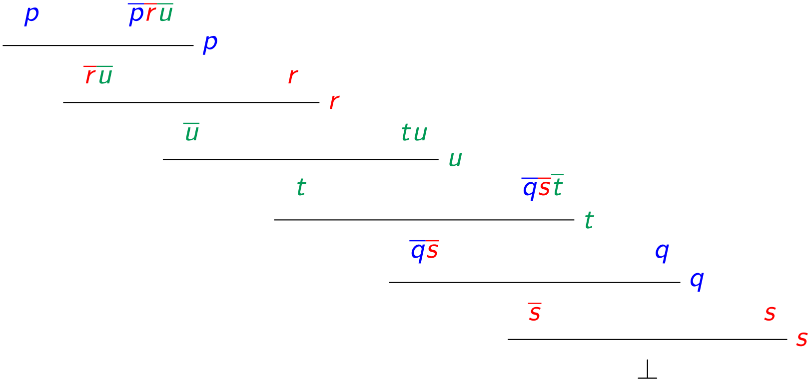
- Proof of unsatisfiability



Interpolant Generation

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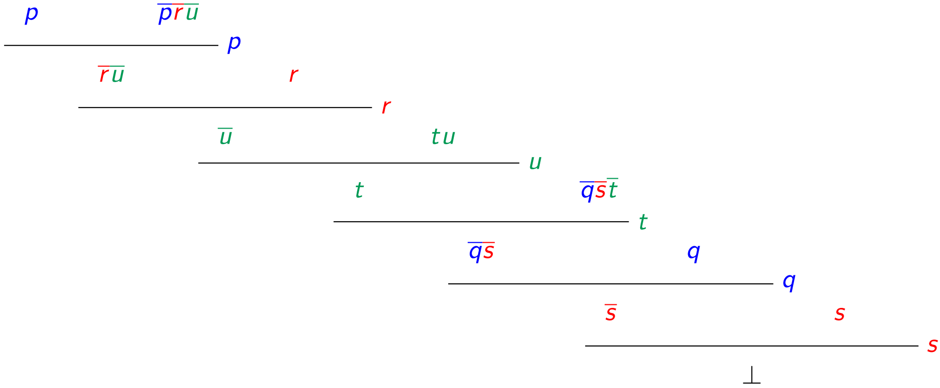

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$$\dfrac{p \ \{\bot\} \qquad \overline{p}\,\overline{r}\,\overline{u}}{\overline{r}\,\overline{u}}\ p$$

$$\dfrac{\overline{r}\,\overline{u} \qquad r}{\overline{u}}\ r$$

$$\dfrac{\overline{u} \qquad tu}{t}\ u$$

$$\dfrac{t \qquad \overline{q}\,\overline{s}\,\overline{t}}{\overline{q}\,\overline{s}}\ t$$

$$\dfrac{\overline{q}\,\overline{s} \qquad q \ \{\bot\}}{\overline{s}}\ q$$

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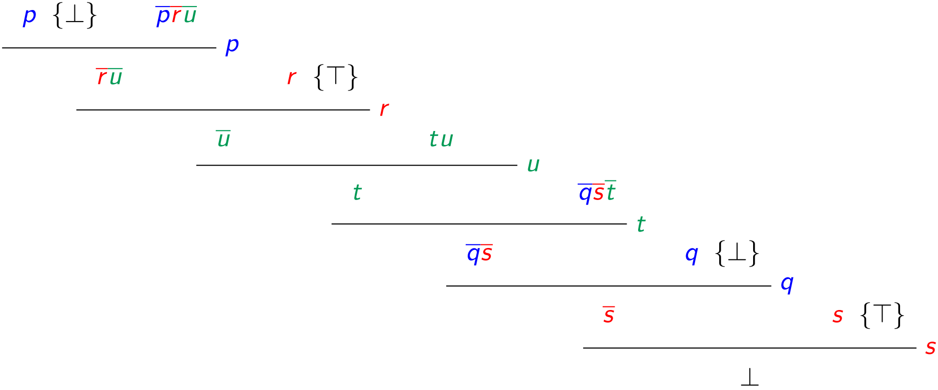

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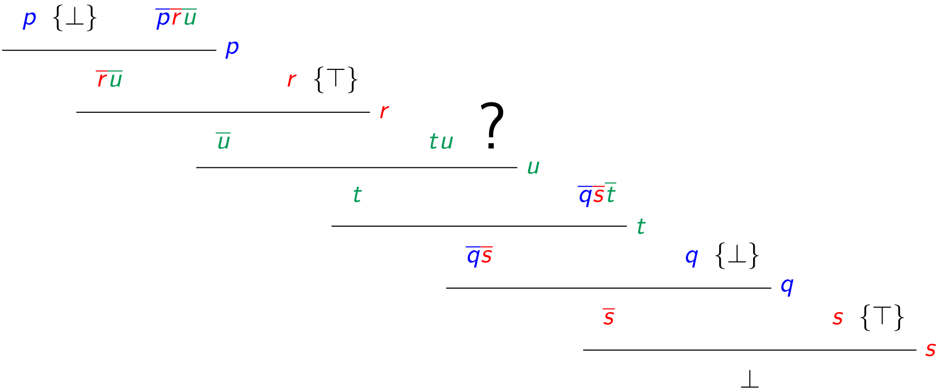

Interpolation
Challenge

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Interpolation

Possible Solutions

- Need for proof not to contain AB-mixed predicates

- Tune solvers to avoid generating AB-mixed predicates [Cimatti08,Beyer08]

- Need for proof not to contain AB-mixed predicates
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- Transform proof to remove AB-mixed predicates

Proof Transformation

Motivation

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- Motivation: standard SMT techniques can require addition of AB-mixed predicates
  - Theory reduction via Lemma on Demand [DeMoura02, Barrett06]

    Reduction of AX to EUF

    Reduction of LIA to LRA

    Ackermann's Expansion
  - Theory combination via DTC [Bozzano05]

Outline

Outline

1. Background
2. Motivation and Related Work
3. Contribution
   - Proof Transformation for Interpolation and Reduction
4. Summary and Future Work

Contribution

Proof Transformation Framework

- Proof rewriting framework based on local rules

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- Isolation of AB-mixed predicates into subtrees

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Contribution

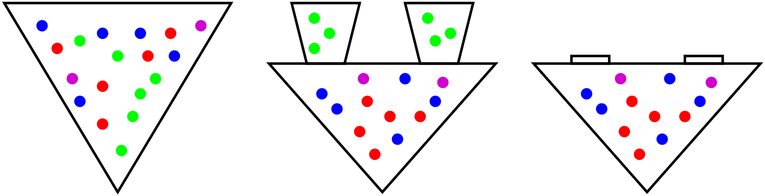
Proof Transformation Framework

- Proof rewriting framework based on local rules
- Isolation of AB-mixed predicates into subtrees
- Removal of AB-mixed subtrees
- No more AB-mixed predicates, proof still valid

(a) Initial proof: A-local, B-local, AB-common, AB-mixed

(b) Transformed proof: AB-mixed predicates isolated into subtrees

(c) Final proof: AB-mixed subtrees removed, new leaves are theory lemmata



(a) (b) (c)

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  - (Partial) interpolant generation for theory (combination) lemmata [Yorsh05]

Proof Transformation Framework

Features

- Local rewriting rules

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- Exhaustiveness up to symmetry

Proof Transformation Framework

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Proof Transformation Framework

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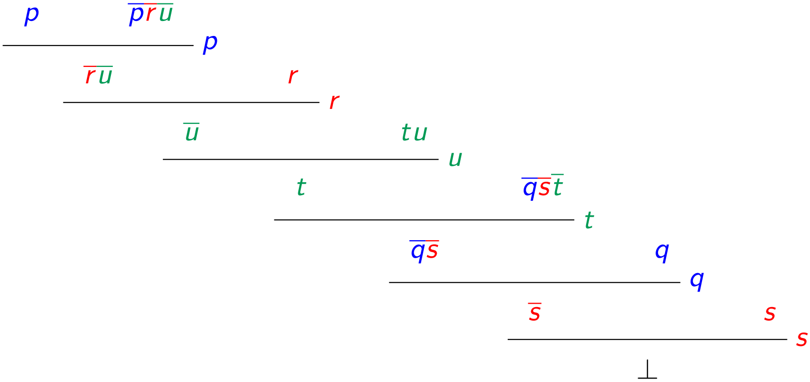
- AB-mixed predicates isolation into subtrees

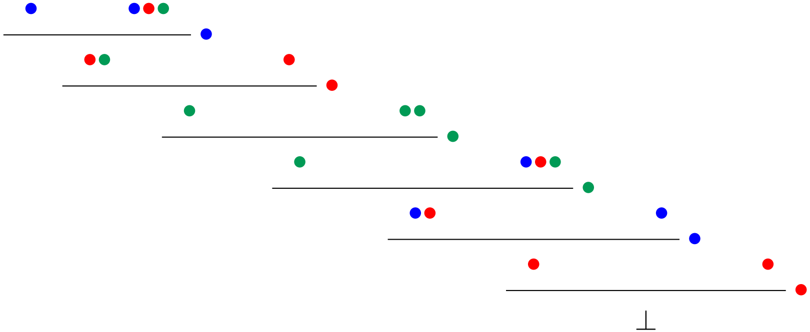
- $A \triangleq \{p, q\}$ $\quad B \triangleq \{r, s\}$ $\quad L \triangleq \{tu, \overline{p}\,\overline{r}\,\overline{u}, \overline{q}\,\overline{s}\,\overline{t}\}$
- Proof of unsatisfiability



Reduction LIA to LRA

Transformation

- Proof of unsatisfiability



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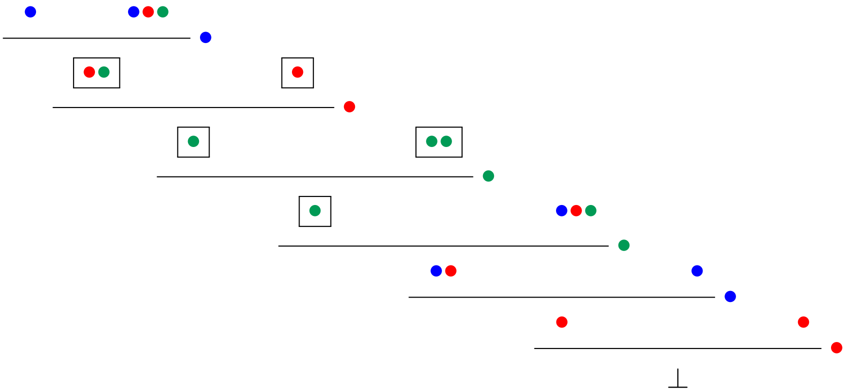

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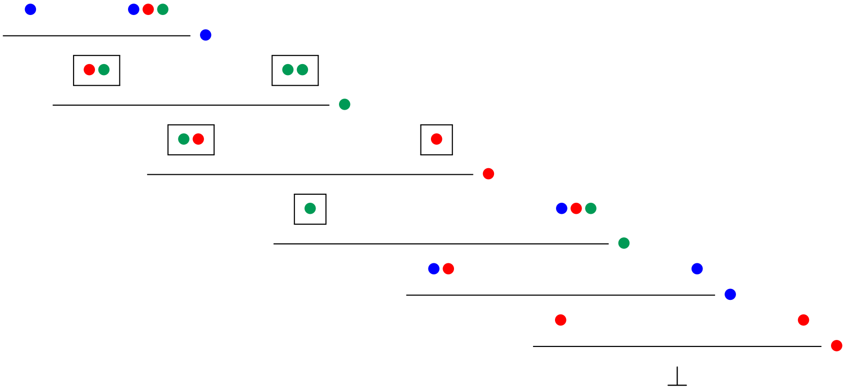

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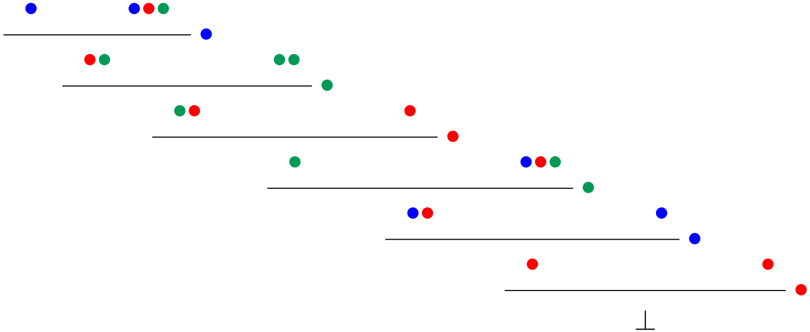

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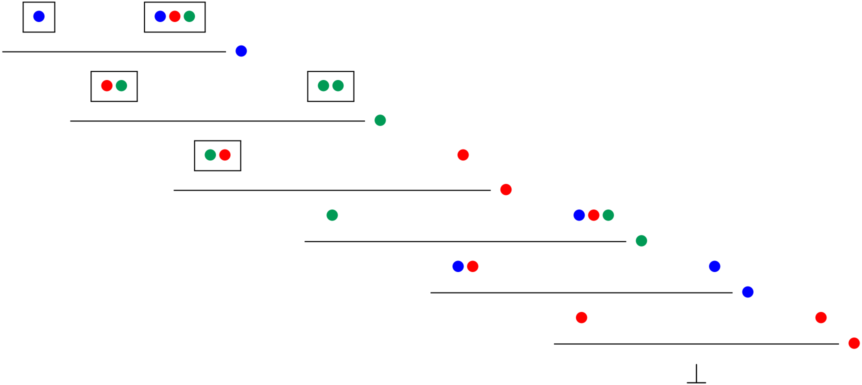

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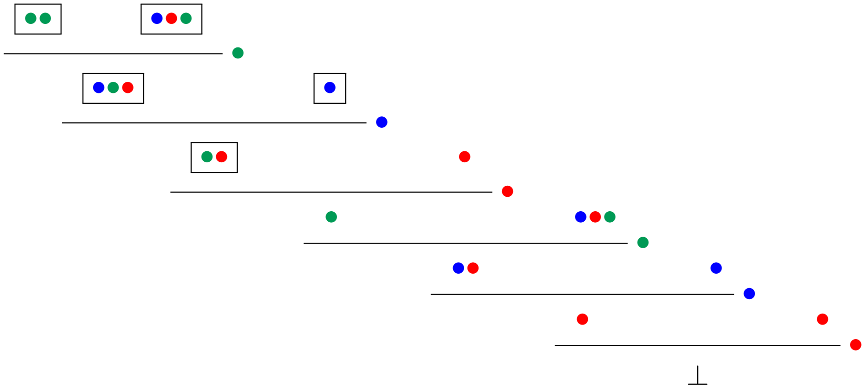

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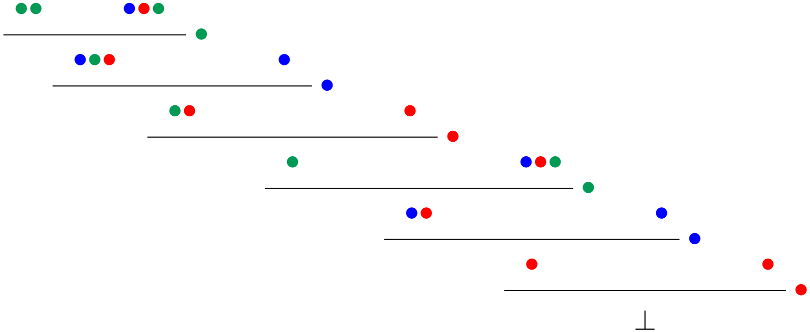

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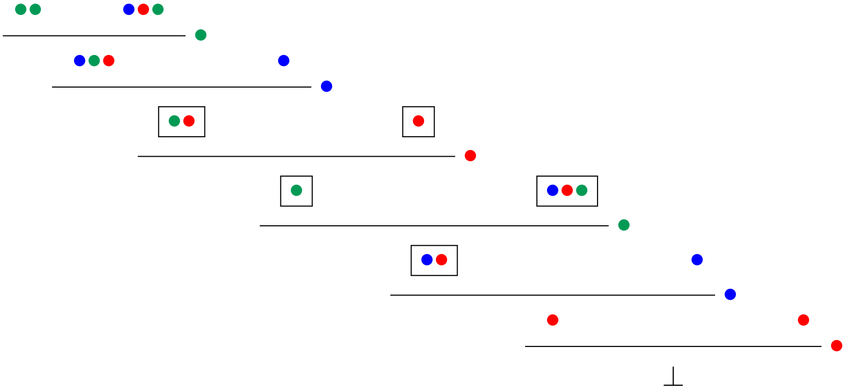

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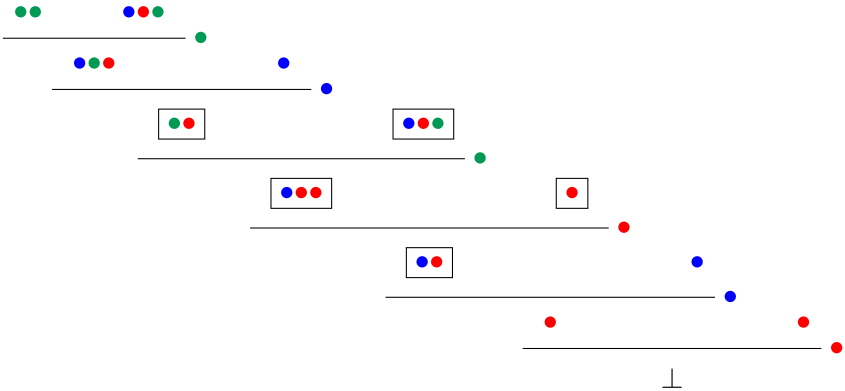

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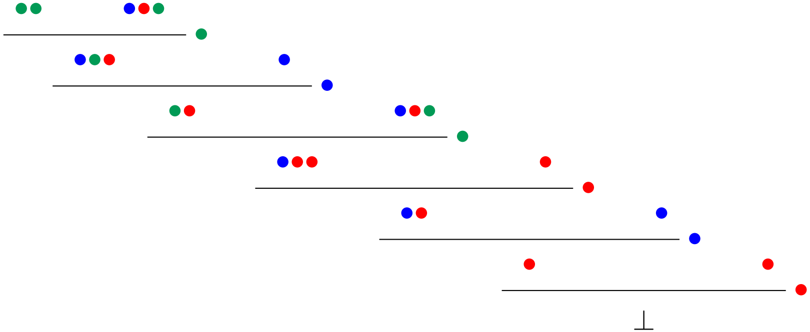

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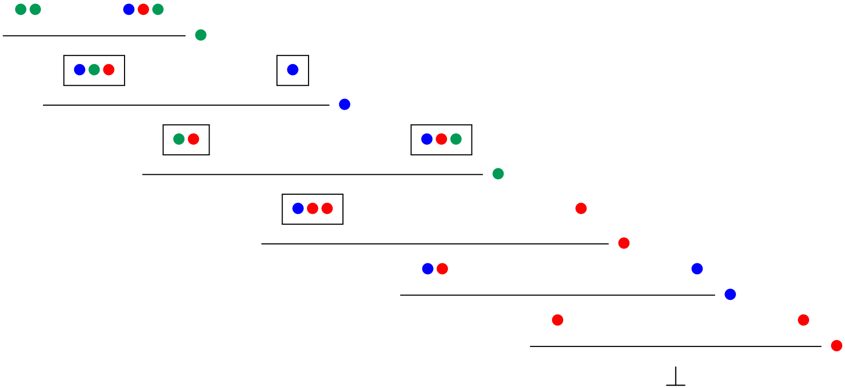

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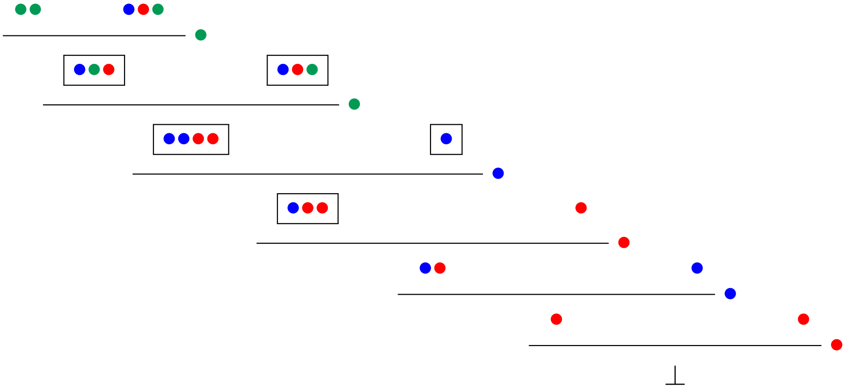

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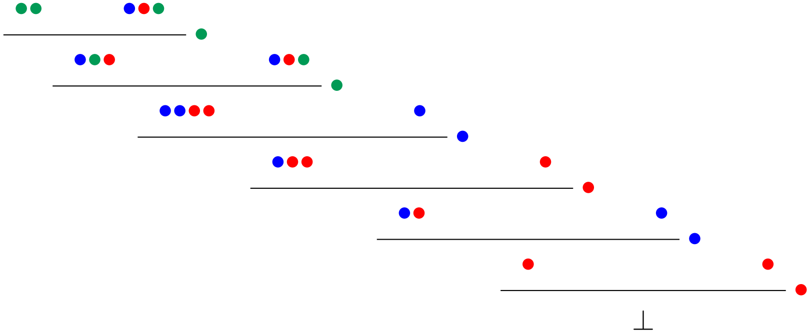

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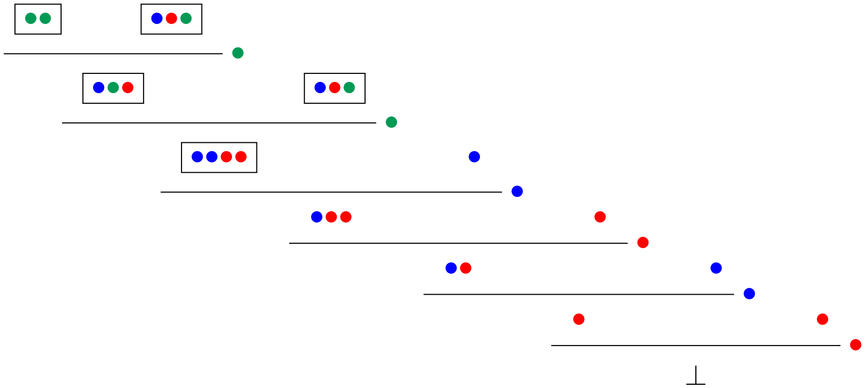

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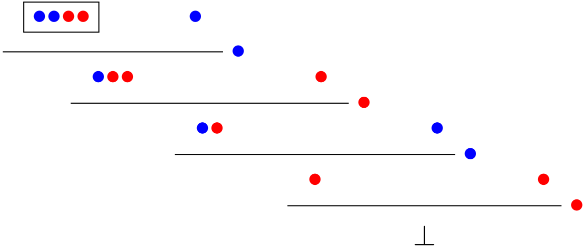

Proof Transformation Framework

Considerations

- Potential drawbacks

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$B1$	$$\dfrac{\dfrac{pqC \qquad \overline{p}qD}{qCD}\,p \qquad p\overline{q}E}{pCDE}\,q \quad \Rightarrow \quad \dfrac{pqC \qquad p\overline{q}E}{pCE}\,q$$

Transformation Framework

Local rewriting rules

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- Redundancy as reintroduction variable after elimination

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Transformation Framework

Local rewriting rules

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- Redundancy as reintroduction variable after elimination
- Subproof simplification
- Subproof root strengthening

Transformation Framework

Local rewriting rules

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$A2$	$\dfrac{\dfrac{pqC \quad \overline{p}D}{qCD}\,p \qquad \overline{q}E}{CDE}\,q \quad \Rightarrow \quad \dfrac{\dfrac{pqC \quad \overline{q}E}{pCE}\,q \qquad \overline{p}D}{CDE}\,p$

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- Pivots swapping
- Topology perturbation
- Redundancies exposure

Local rewriting rules

$A1$	$\dfrac{\dfrac{pqC \qquad \overline{p}qD}{qCD}\,p \qquad \overline{q}E}{CDE}\,q$	$\Rightarrow$	$\dfrac{\dfrac{pqC \qquad \overline{q}E}{pCE} \qquad \dfrac{\overline{q}E \qquad \overline{p}qD}{\overline{p}DE}\,q}{CDE}\,p$
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$B2'$	$\dfrac{\dfrac{pqC \qquad \overline{p}D}{qDC}\,p \qquad p\overline{q}E}{pCDE}\,q$	$\Rightarrow$	$\dfrac{pqC \qquad p\overline{q}E}{pCE}\,q$
$B3$	$\dfrac{\dfrac{pqC \qquad \overline{p}D}{qCD}\,p \qquad \overline{p}\overline{q}E}{\overline{p}CDE}\,q$	$\Rightarrow$	$\overline{p}D$

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Framework and Benchmarks

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    - Academic and industrial problems

Evaluation

Experimental results over QF_UFIDL

Group	#	$\#AB$	$\%_{time}$	$\%_{nodes}$	$\%_{edges}$
RDS	2	7	93%	2%	2%
EufLaAr	2	103	91%	30%	26%
pete	6	4	33%	8%	9%
pete2	56	17	59%	27%	32%
uclid	8	11	64%	37%	42%
Overall	74	17	59%	26%	30%

- # — number of benchmarks solved
- $\#AB$ — average number of AB-mixed predicates in proof
- $\%_{time}$ — average time overhead
- $\%_{nodes}$, $\%_{edges}$ — average difference in proof size

Comparison

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- Rule-based approach
  - **Pros**
    Flexibility in rules application
    Flexibility in amount of transformation
    Can expose redundancies
  - **Cons**
    Local information

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Reduction Algorithm

- Based on a sequence of proof traversals (e.g. topological order)

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- Parameterized in number of traversals and time limit
- Examination non-leaf clauses
  - Pivot in both antecedents $\longrightarrow$ update, match context, apply rule

$$\frac{qC'D' \qquad \overline{q}E'}{CDE}\,q \quad \Rightarrow \quad \frac{qC'D' \qquad \overline{q}E'}{C'D'E'}\,q \quad \Rightarrow \quad \frac{\dfrac{pqC' \qquad \overline{p}D'}{qC'D'}\,p \qquad \overline{q}E'}{C'D'E'}\,q$$

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- Easy combination with RecyclePivots

- Implemented in C++ and integrated with OpenSMT
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Combined Approach Evaluation

Experimental results over SMT: QF_UF, QF_IDL, QF_LRA, QF_RDL

	#	Avg_{nodes}	Avg_{edges}	Avg_{core}	$T(s)$	Max_{nodes}	Max_{edges}	Max_{core}
RP	1370	6.7%	7.5%	1.3%	1.7	65.1%	68.9%	39.1%
Ratio								
0.01	1366	8.9%	10.7%	1.4%	3.4	66.3%	70.2%	45.7%
0.025	1366	9.8%	11.9%	1.5%	3.6	77.2%	79.9%	45.7%
0.05	1366	10.7%	13.0%	1.6%	4.1	78.5%	81.2%	45.7%
0.075	1366	11.4%	13.8%	1.7%	4.5	78.5%	81.2%	45.7%
0.1	1364	11.8%	14.4%	1.7%	5.0	78.8%	83.6%	45.7%
0.25	1359	13.6%	16.6%	1.9%	7.6	79.6%	84.4%	45.7%
0.5	1348	15.0%	18.4%	2.0%	11.5	79.1%	85.2%	45.7%
0.75	1341	16.0%	19.5%	2.1%	15.1	79.9%	86.1%	45.7%
1	1337	16.7%	20.4%	2.2%	18.8	79.9%	86.1%	45.7%

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- Avg_{nodes}, Avg_{edges}, Avg_{core} — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- Max_{nodes}, Max_{edges}, Max_{core} — max reduction in proof size

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Combined Approach Evaluation

Experimental results over SMT: QF_UF, QF_IDL, QF_LRA, QF_RDL

	#	Avg_{nodes}	Avg_{edges}	Avg_{core}	$T(s)$	Max_{nodes}	Max_{edges}	Max_{core}
RP	1370	6.7%	7.5%	1.3%	1.7	65.1%	68.9%	39.1%
Ratio								
0.01	1366	8.9%	10.7%	1.4%	3.4	66.3%	70.2%	45.7%
0.025	1366	9.8%	11.9%	1.5%	3.6	77.2%	79.9%	45.7%
0.05	1366	10.7%	13.0%	1.6%	4.1	78.5%	81.2%	45.7%
0.075	1366	11.4%	13.8%	1.7%	4.5	78.5%	81.2%	45.7%
0.1	1364	11.8%	14.4%	1.7%	5.0	78.8%	83.6%	45.7%
0.25	1359	13.6%	16.6%	1.9%	7.6	79.6%	84.4%	45.7%
0.5	1348	15.0%	18.4%	2.0%	11.5	79.1%	85.2%	45.7%
0.75	1341	16.0%	19.5%	2.1%	15.1	79.9%	86.1%	45.7%
1	1337	16.7%	20.4%	2.2%	18.8	79.9%	86.1%	45.7%

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- Avg_{nodes}, Avg_{edges}, Avg_{core} — average reduction in proof size
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- Max_{nodes}, Max_{edges}, Max_{core} — max reduction in proof size

Combined Approach Evaluation

Experimental results over SAT

	#	Avg_{nodes}	Avg_{edges}	Avg_{core}	$T(s)$	Max_{nodes}	Max_{edges}	Max_{core}
RP	25	5.9%	6.5%	1.7%	10.8	33.1%	33.4%	30.3%
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0.05	25	7.0%	8.2%	1.8%	40.0	34.0%	34.4%	30.5%
0.075	25	7.2%	8.4%	1.8%	49.3	34.7%	35.1%	30.5%
0.1	25	7.3%	8.4%	1.8%	60.2	34.7%	35.1%	30.5%
0.25	25	7.6%	8.8%	1.9%	125.3	39.8%	40.6%	31.7%
0.5	25	7.8%	9.1%	1.9%	243.5	41.0%	41.9%	32.1%
0.75	25	7.9%	9.3%	1.9%	360.0	41.6%	42.6%	32.1%
1	23	8.4%	9.9%	2.1%	175.6	33.1%	33.4%	30.6%

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Outline

Summary

- Proof transformation

  1. Interpolation, SMT, AB-mixed predicates

Summary

- Proof transformation

  1. Interpolation, SMT, AB-mixed predicates

  2. Proof transformation framework for AB-mixed predicates removal

Summary

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  2. Proof transformation framework for AB-mixed predicates removal
  3. Easy combination:
     - Standard SMTs
     - State-of-the art interpolant generation procedures

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     - State-of-the art interpolant generation procedures
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Future Work

- Exploitation of DPLL proof structure

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- Evaluation on concrete applications (e.g. interpolation)

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- Exploitation of DPLL proof structure
- Evaluation on concrete applications (e.g. interpolation)
- Rule-based control of interpolants' strength

Publications

- Proof reduction

S.F. Rollini, R. Bruttomesso and N. Sharygina
An Efficient and Flexible Approach to Resolution Proof Reduction.
HVC 2010.

- Proof manipulation for interpolation

R. Bruttomesso, S.F. Rollini, N. Sharygina and A. Tsitovich
Flexible Interpolation with Local Proof Transformations.
ICCAD 2010

Thanks for your attention!

http://www.verify.inf.usi.ch/