

# Outline

- Not merely "complexity, networks, abstraction, recursion, modularity,...."
- But very specific forms of these.
- Formal methods have great potential
- Illustrate with case studies and cartoons: Internet versus bacterial biosphere
- Implicitly: importance of formal methods, not merely modeling and simulation



# Network Math and Engineering (NetME) Challenges

- Predictive modeling, simulation, and analysis of complex systems in technology and nature
- Theoretical foundation for design of network architectures
- Balance rigor/relevance, integrative/coherent
- Model/simulate is critical but limited
  - Predicting rare but catastrophic events
  - Design, not merely analysis
  - Managing complexity and uncertainty

# "Architecture"

- Most persistent, ubiquitous, and global features of organization
- Constrains what is possible for good or bad
- Platform that enables (or prevents) innovation, sustainability, etc,
- Internet, biology, energy, manufacturing, transportation, water, food, waste, law, etc
- Existing architectures are unsustainable
- Theoretical foundation is fragmented, incoherent, incomplete

## **Stochastics in Biology**

- Arkin, Gillespie, Petzold, Khammash, El-Samad, Munsky, Paulsson, Vinnicombe, many others...
- Noise in the cellular environment
  - Elowitz, van Oudenaarden, Collins, Swain, Xie, Elston, ...
- Stochastic Monte Carlo Simulation
  - Kurtz, Gibson, Bruck, Anderson, Rathinam, Cao, Salis, Kaznessis, ...
- Statistical moment computations
  - Hespanha, Singh, Verghese, Gomez-Uribe, Kimura
- Density function computations
  - McNamara, Sidje, ...
- Stochastic differential equation approximations
  - van Kampen, Kurtz, Elf, Ehrenberg,...
- Spatial stochastic models and tools
  - Elf, Iglesias,...

#### Very incomplete, idiosyncratic list

# **Other Influences**

- Internet (Kelly/Low, Willinger, Clark, Wroclawski, Day, Chang, etc etc)
- Biology/Medicine (Savageau, G&K, Mattick, Csete, Arkin, Alon, Caporale, de Duve, Exerc Physio, Acute Care, etc etc...)
- Architecture (Alexander, Salingeros,...)
- Aerospace (many, Maier is a good book)
- Philosophy/History (Fox Keller, Jablonka&Lamb)
- Physics/ecology (Carlson)
- Management (Baldwin,...)
- Resilience/Safety/Security Engineering/Economics (Wood, Anderson, Leveson, ...)

# Biology versus the Internet

## Similarities

- Evolvable architecture
- Robust yet fragile
- Constraints/deconstrain
- Layering, modularity
- Hourglass with bowties
- Feedback
- Dynamic, stochastic
- Distributed/decentralized
- *Not* scale-free, edge-of-chaos, self-organized criticality, etc

## Differences

- Metabolism
- Materials and energy
- Autocatalytic feedback
- Feedback complexity
- Development and regeneration
- >4B years of evolution
- How the parts work?

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# Focus on bacterial biosphere

## **Question: Human complexity**

## Robust

- ③ Efficient, flexible metabolism
- © Regeneration & renewal
- ③ Rich microbial symbionts
- Immune systems
- Complex societies
- Advanced technologies

## Yet Fragile

- $\ensuremath{\mathfrak{S}}$  Obesity and diabetes
- $\ensuremath{\mathfrak{S}}$  Cancer
- ③ Parasites, infection
- ⊗ Inflammation, Auto-Im.
- Epidemics, war, ...

## **Mechanism?**

## Robust

- ③ Efficient, flexible metabolism
- ③ Regeneration & renewal
  - Sat accumulation
  - Insulin resistance
  - Inflammation

# Fluctuating energy

## Yet Fragile

- $\ensuremath{\mathfrak{S}}$  Obesity and diabetes
- S Cancer
  - S Fat accumulation
  - ℬ Insulin resistance
  - $\ensuremath{\mathfrak{S}}$  Inflammation

## Static energy

#### Implications/ Generalizations abolism

#### Robust

- © Efficient, flexible metabolism
- © Rich microbial symbionts
- Immune systems
- © Regeneration & renewal
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#### Yet Fragile

- Obesity and diabetes
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- S Cancer
- Epidemics, war, ...
- ▲ Catastrophic failures
- Fragility = Hijacking, side effects, unintended...
  of mechanisms evolved for robustness
- Complexity is driven by control, robust/fragile tradeoffs
- Math: New robust/fragile conservation laws



## **Non-networked Systems**



Resources

## **Network requirements**





Geographically local

## Layered solution



#### Constraints

## **Universal control**

## That deconstrain



That deconstrain

















## Layered solution



## How many layers are there?



As many as you need.

## Layered solution



#### And layers have sublayers



## Layered solution









Ancient network architecture: "Bell-heads versus Net-heads" Layers (Net) Computer



Pathways (Bell) Communications

Phone systems

## **Cyber-Physical Theories**

- Thermodynamics
- Communications
- Control
- Computation

# Cyber



- Thermodynamics
- Communications
- Control
- Computation

- Thermodynamics
- Communications
- Control
- Computation

## Internet

## Bacteria

### **Case studies**





- Thermodynamics
- Communications
- Control
- Computation

- Thermodynamics
- Communications
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## Promising unifications
Theoretical framework: Constraints that deconstrain



- Optimization
- Optimal control
- Robust control
- Game theory
- Network coding

Architecture is *not* graph topology.



Architecture facilitates arbitrary graphs.

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DNA - RNA - Protein

# Network architecture?





## Layered Brain (Hawkins)?







## Inside every cell



#### Core metabolic bowtie























If we drew the feedback loops the

















That deconstrain



## Running only the top layers



Mature red blood cells live 120 days

> "metabolism first" origins of life?

Reactions

**Flow/error** 

**Protein level** 

Reactions

**Flow/error** 

**RNA** level

Reactions Flow/error

**DNA level** 



## DNA







# Top to bottom

- Metabolically costly but fast to cheap but slow
- Special enzymes to general polymerases
- Allostery to regulated recruitment
- Analog to digital
- High molecule count to low (noise)

### **Rich Tradeoffs**





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Autocatalytic feedback



What theory is relevant to these more complex feedback systems?















## "Central dogma"

**RNAp** 

# **Protein**





### This is just charging and discharging















## Recursive control structure





## Recursive control structure

**Physical** 



Bewildering w/out clear grasp of layered architecture

#### **Horizontal gene transfer**

Eukaryotes

Fungi

**Plants** 

Algae

Animals

Archaea

Bacteria that gave rise to chloroplasts

Bacteria that gave rise to mitochondria

orarchaeota

#### HGT and Shared Protocols

Bacteria

What is locus of early evolution?

Architecture!?!

Hyperthermophilic bacteria

Common Ancestral Community of Primitive Cells



#### **Horizontal gene transfer**





fan-in of diverse inputs fan-out of diverse outputs

Diverse **Highly robust** function Diverse Evolvable Deconstrained Diverse Robust Constraints that yet fragile deconstrain <u>components</u>

Universal Control

- Highly fragile
- Universal
- Frozen

universal

carriers

- Constrained
- Hijacking



What theory is relevant to these more complex feedback systems?













### **Autocatalytic**



### Control



Caution: mixed cartoon

$$S \quad j\omega = rac{X \quad j\omega}{U \quad j\omega}$$

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S\right| \, j\omega \, \left|d\omega \ge 0\right|$$

$$\int_{-\infty}^{\infty} \ln \left| S \quad j\omega \right| d\omega = \int_{-\infty}^{\infty} \ln \left| \frac{X \quad j\omega}{U \quad j\omega} \right| d\omega$$

$$= \int_{-\infty}^{\infty} \ln |X| \, j\omega \, |d\omega - \int_{-\infty}^{\infty} \ln |U| \, j\omega \, |d\omega$$

Entropy rates



#### **Hard limits**






[a system] can have [a property] *robust* for [a set of perturbations]

Yet be *fragile for* 

[a different property]

Or [a different perturbation]



### Robust yet fragile = fragile robustness

$$S \quad j\omega = \frac{X \quad j\omega}{U \quad j\omega}$$

### **Hard limits**





$$S \quad j\omega = rac{X \quad j\omega}{U \quad j\omega}$$

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S\right| j\omega \left|d\omega \ge 0\right|$$

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S \quad j\omega\right| \frac{z}{z^{2}+\omega^{2}}d\omega \ge \ln\left|\frac{z+p}{z-p}\right|$$



The plant can make this tradeoff worse.

$$S \quad j\omega = rac{X \quad j\omega}{U \quad j\omega}$$

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S\right|\,j\omega\,\left|d\omega\geq0\right|$$

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S \quad j\omega\right| \frac{z}{z^{2}+\omega^{2}}d\omega \ge \ln\left|\frac{z+p}{z-p}\right|$$

All controllers:  $\geq$  Biological cells: =



$$z = \frac{k}{q}$$
  $p = RHPzero s^2 + q\alpha + k s - \alpha k$ 

$$S \quad j\omega = \frac{X \quad j\omega}{U \quad j\omega}$$

$$\frac{1}{\pi} \int_{0}^{\infty} \ln \left| S \quad j\omega \right| d\omega \ge 0$$
$$\frac{1}{\pi} \int_{0}^{\infty} \ln \left| S \quad j\omega \right| \frac{z}{z^{2} + \omega^{2}} d\omega \ge \ln \left| \frac{z + p}{z - p} \right|$$

Small *z* is *bad*.





 $p = RHPzero s^2 + q\alpha + k s - \alpha k$ 

# Small *z* is *bad* (oscillations and crashes)

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S \quad j\omega\right| \frac{z}{z^{2}+\omega^{2}}d\omega \ge \ln\left|\frac{z+p}{z-p}\right|$$

### Small z =

- small k and/or
- large q



Correctly predicts conditions with "glycolytic oscillations"

$$S \quad j\omega = rac{X \quad j\omega}{U \quad j\omega}$$

$$\frac{1}{\pi}\int_{0}^{\infty}\ln\left|S\right| \, j\omega \, \left|d\omega \ge 0\right|$$

$$\int_{-\infty}^{\infty} \ln \left| S \quad j\omega \right| d\omega = \int_{-\infty}^{\infty} \ln \left| \frac{X \quad j\omega}{U \quad j\omega} \right| d\omega$$

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Entropy rates



#### **Hard limits**











### Network motifs in the transcriptional regulation network of *Escherichia coli*

Shai S. Shen-Orr<sup>1</sup>, Ron Milo<sup>2</sup>, Shmoolik Mangan<sup>1</sup> & Uri Alon<sup>1,2</sup>



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## mRNA activity is actively controlled.









The greatest complexity here is primarily in the control of *rates* 



Gly G1P G6P F6P F1-6B Gly3 ATP 13BPG 3PG Oxa 2PG PEP Pyr ACA TCA JADH Cit

That is not always the case.







### All at the DNA layer





#### Network motifs in the transcriptional regulation network of *Escherichia coli*






















